Topological Distance Between Nonplanar Transportation Networks

Ahmed Abdelkader[1], Geoff Boeing[2],
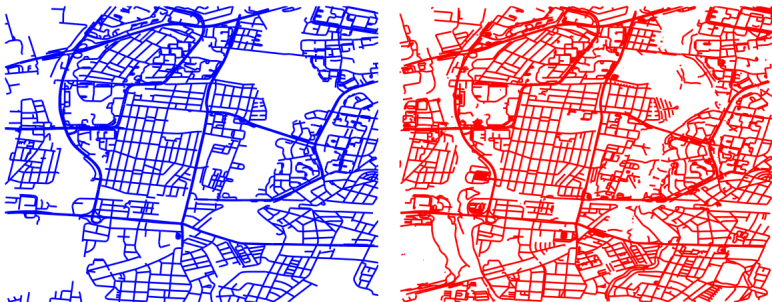Brittany Terese Fasy[3], David Millman[3]

[1]University of Maryland, College Park

[2]Northeastern University

[3]Montana State University

Fall Workshop on Computational Geometry
Queens College, City University of New York
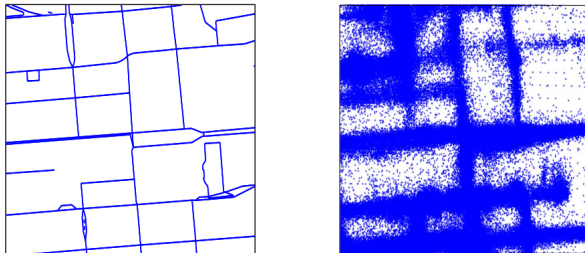Flushing, NY
October 26th, 2018

Map Comparison

Two maps of Berlin: 2007 (left) vs. 2013 (right)
[Ahmad, Fasy and Wenk, SigSpatial'14]

Problem

How to compare two such maps?
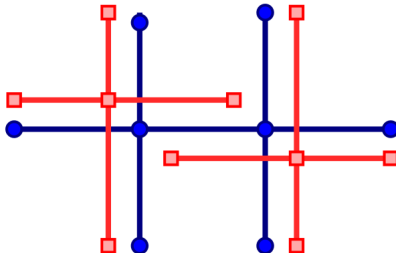
Why Map Comparison?

Ground truth vs. density plot [AFW, SigSpatial'15]

Applications

- Evaluation and comparison of map reconstruction algorithms
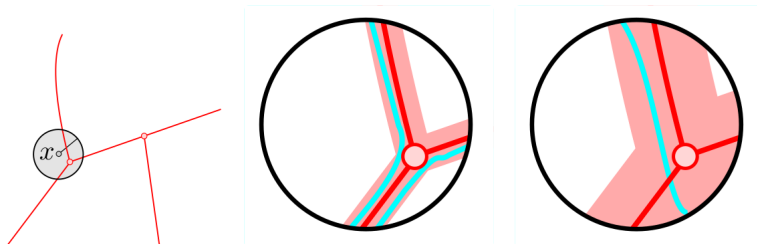- Urban morphology

Why Topology?

Two street intersections [AFW, SigSpatial'14]

Two types of error

- Hausdorff distance: low
- Topology: different

Topology-based Map Comparison

[AFW, SigSpatial'14]

A topological signature at x

- Compute the *local persistent homology* in the neighborhood of x
    - Define *offsets* by thickening street segments
    - Compute a filtration from overlapping offsets
    - Compute the persistent homology of the filtration
    - .. relative to the boundary of the neighborhood
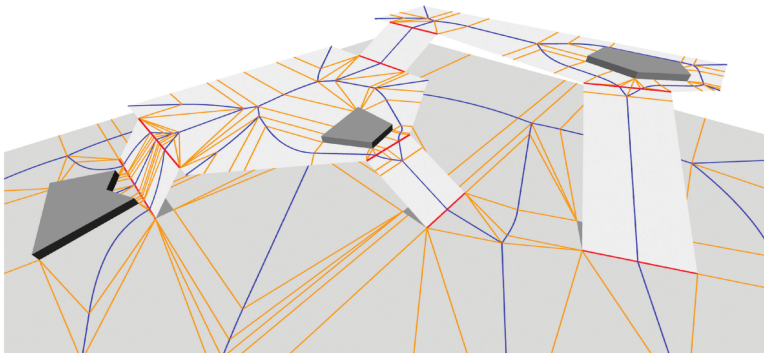- Compare the barcodes for different maps

Assumption of Planarity ..

.. can lead to drastic errors in street network analysis
[Boeing, Environment & Planning B '18]

Left: High Five Interchange, Dallas, USA
Right: Magdeburg Water Bridge, Magdeburg, Germany
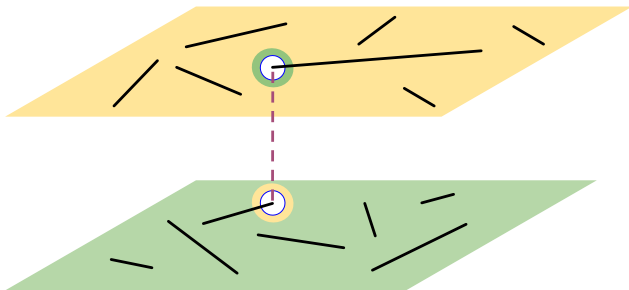
Multi-Layered Environments

[van Toll, Cook, van Kreveld and Geraerts, TSAS'18]

Nearly 2D

- For many planning problems, 3D is an overkill
- Extend algorithms and data structures from 2D
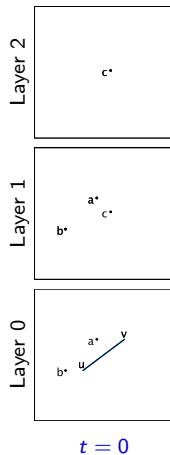
Grade-separated Street Networks

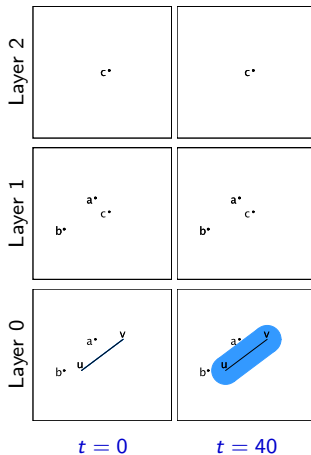Abstract model for street networks

Allowing non-planarity

- Multiple copies of $\mathbb{R}^2$ (*layers*) identified at special vertices (*portals*)
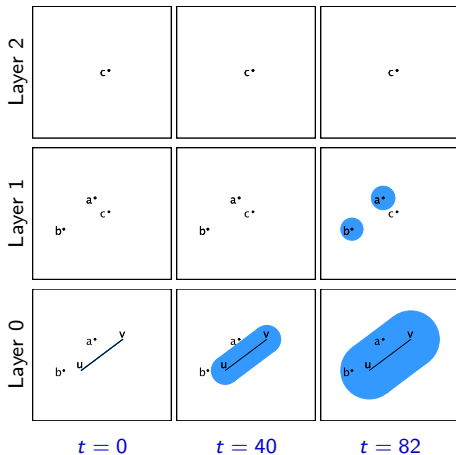- Segment offsets grow through portals onto connected layers

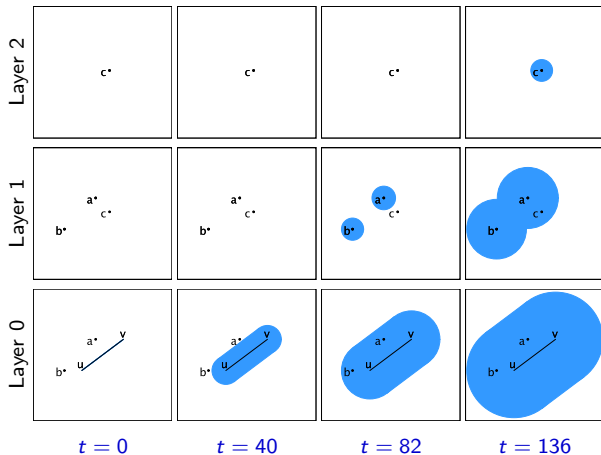# Segment Offsets

$t = 0$

Offset of a segment $\overline{uv}$ across three layers

Segment Offsets

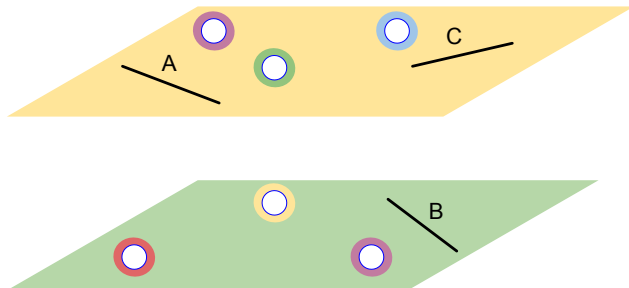Offset of a segment $\overline{uv}$ across three layers

Segment Offsets

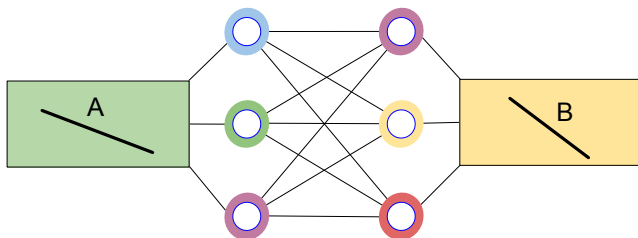Offset of a segment $\overline{uv}$ across three layers

Segment Offsets

Offset of a segment $\overline{uv}$ across three layers

Overlap Computation - Two-way

Determine the earliest time when offsets intersect

- Case(1): segments in the same layer, e.g., A & C $\implies$ old school
- Case(2): segments in different layers, e.g., A & B $\implies$ via portals ..

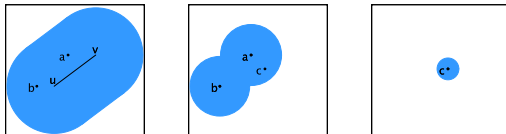# Overlap Computation - Two-way: Case(2)

Shortest path through portal graph

- Precompute all-pairs shortest paths on a *portal graph*
- Connect A and B to portals on respective layers
- Find the shortest path from A to B

Overlap Computation - Three-way

Which layer realizes the overlap?

- Same as segment $\implies$ offset is a segment offset (S)
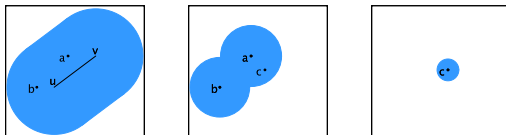- Different layer $\implies$ offset is a disk centered at some portal (D)

Earliest time when offsets intersect .. but at which layer?

- Case(1): SSS $\implies$ segment Voronoi diagram
- Case(2): SSD $\implies$ disk + segment bisector (straight & parabolic arcs)
- Case(3): SDD $\implies$ weighted segment + disk bisector (hyperbola)
- Case(4): DDD $\implies$ Apollonius diagram

Overlap Computation - Three-way
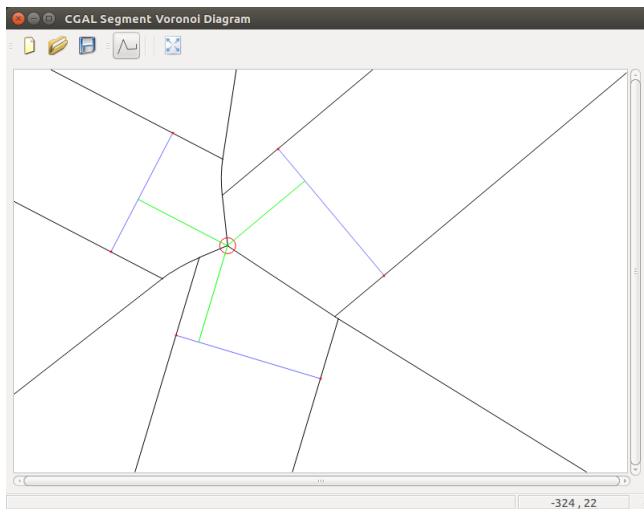
Which layer realizes the overlap?

- Same as segment $\implies$ offset is a segment offset (S)
- Different layer $\implies$ offset is a disk centered at some portal (D)

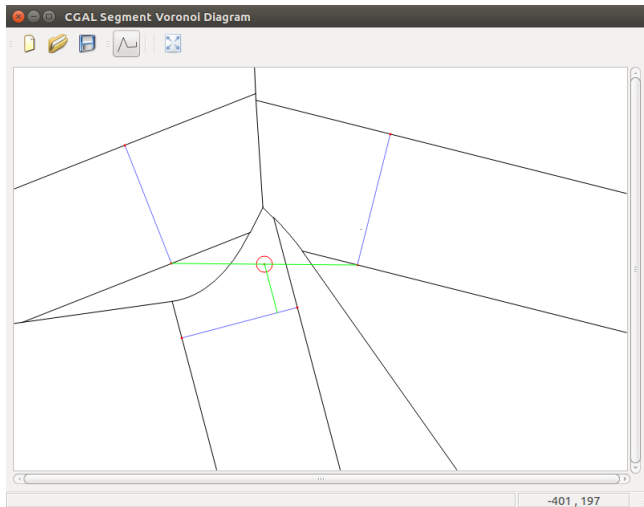**Earliest time when offsets intersect .. but at which layer?**

- Case(1): SSS $\implies$ segment Voronoi diagram
- Case(2): SSD $\implies$ disk + segment bisector (straight & parabolic arcs)
- Case(3): SDD $\implies$ weighted segment + disk bisector (hyperbola)
- Case(4): DDD $\implies$ Apollonius diagram

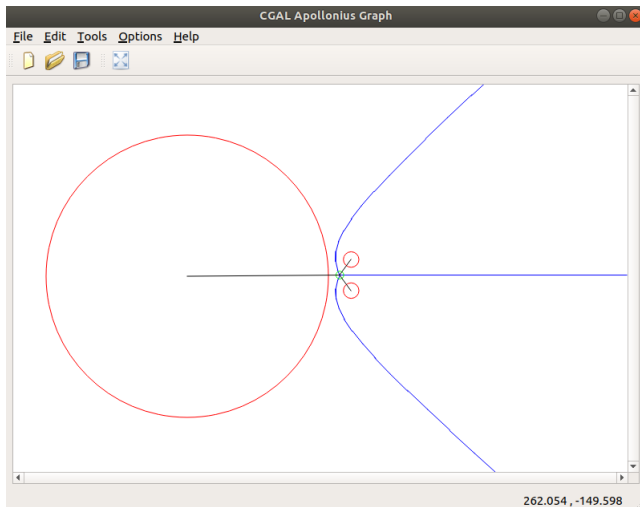# Overlap Computation - Three-way: Case(1) SSS

Case(1.1): earliest overlap at a Voronoi vertex

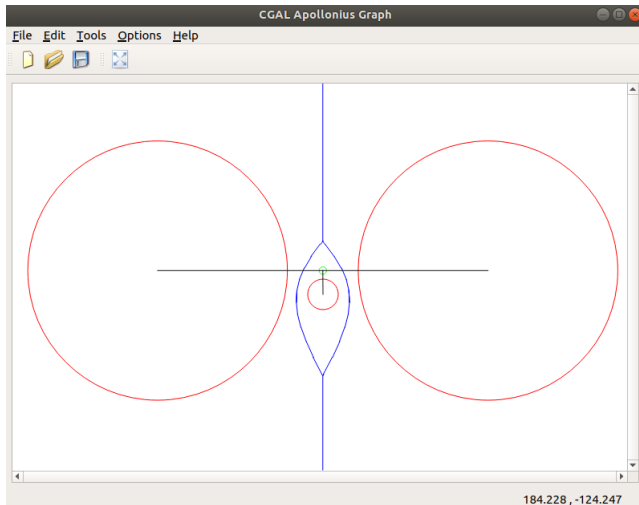# Overlap Computation - Three-way: Case(1) SSS

Case(1.2): earliest overlap in the interior of a cell

Overlap Computation - Three-way: Case(4) DDD

Case(4.1): earliest overlap at an Apollonius vertex

Overlap Computation - Three-way: Case(4) DDD

Case(4.2): earliest overlap in the interior of a cell

Topological Distance for Non-planar Street Networks

- Model grade-separated networks as multi-layered graphs
- Define segment offsets and filtrations across layers
- Next steps
  - Implement LPH computation (based on `CGAL` & `Dionysus`)
  - Run experiments, e.g., data from `OpenStreetMap`
  - Formalize the new map distance

Thanks for listening

Questions?
akader@cs.umd.edu