The computational power of quantum walk

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Why quantum computing?

Fast algorithms for classically hard problems

Fast algorithms for classically hard problems

Fast algorithms for classically hard problems

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Fast algorithms for classically hard problems

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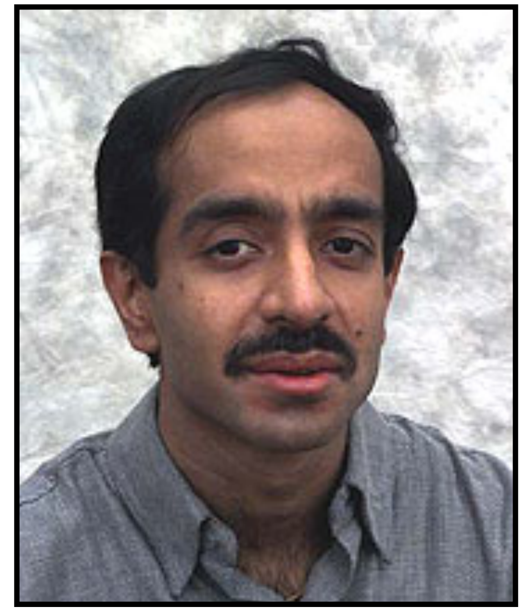

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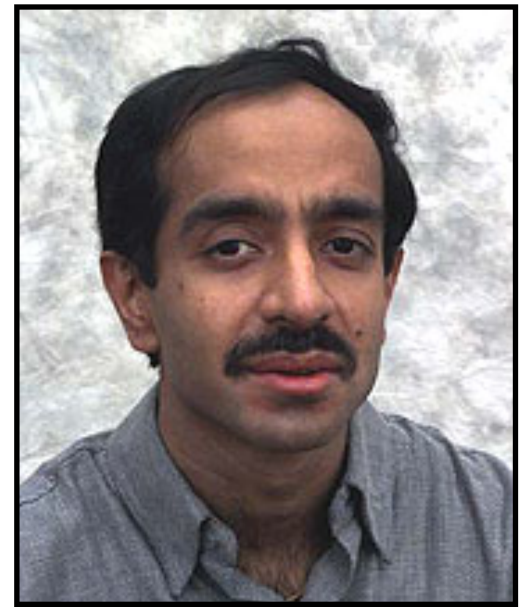

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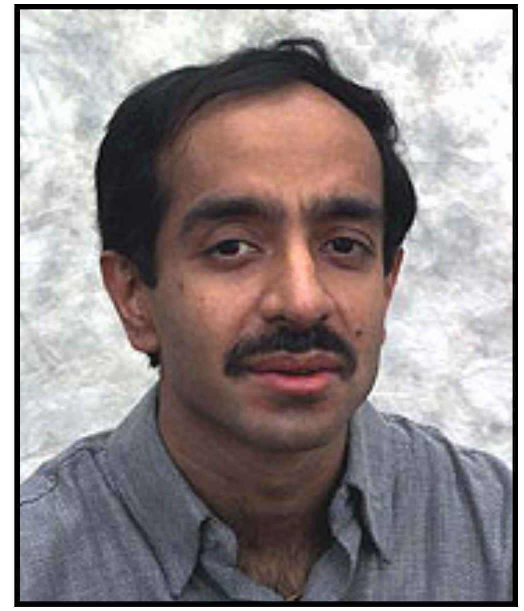

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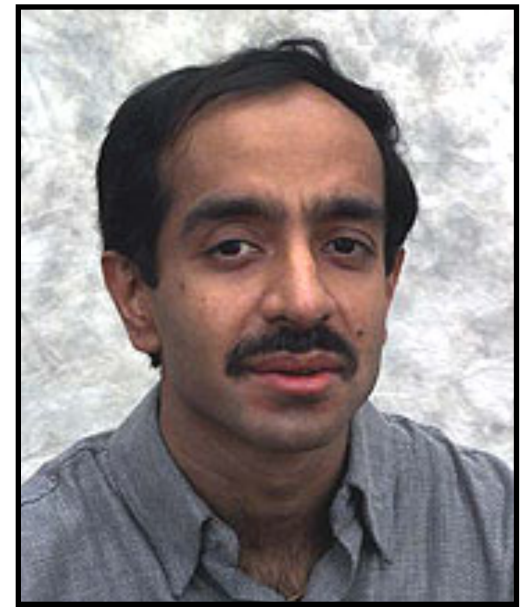

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- Computing discrete logarithms
- Decomposing Abelian groups
- Computations in number fields
- Approximating Gauss sums
- Shifted Legendre symbol
- Counting points on algebraic curves
- Approximating the Jones polynomial (and other topological invariants)
- Simulating quantum systems
- Linear systems
- Computing effective resistance
- …

- Formula evaluation
- Collision finding (k-distinctness, k-sum, etc.)
- Minimum spanning tree, connectivity, shortest paths, bipartiteness of graphs
- Network flows, maximal matchings
- Finding subgraphs
- Minor-closed graph properties
- Property testing (distance between distributions, bipartiteness/expansion of graphs, etc.)
- Checking matrix multiplication
- Group commutativity
- Subset sum
- …

Post-quantum cryptography

Much of the cryptography in use today (e.g., RSA, elliptic curves) could be broken by a quantum computer

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One possible reaction:

Another reaction: Try to understand what quantum computers are good at so we can design cryptosystems they can't break

What can be computed efficiently?

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Main goal of my research: Understand the advantages of quantum over classical computation

The origin of quantum speedup

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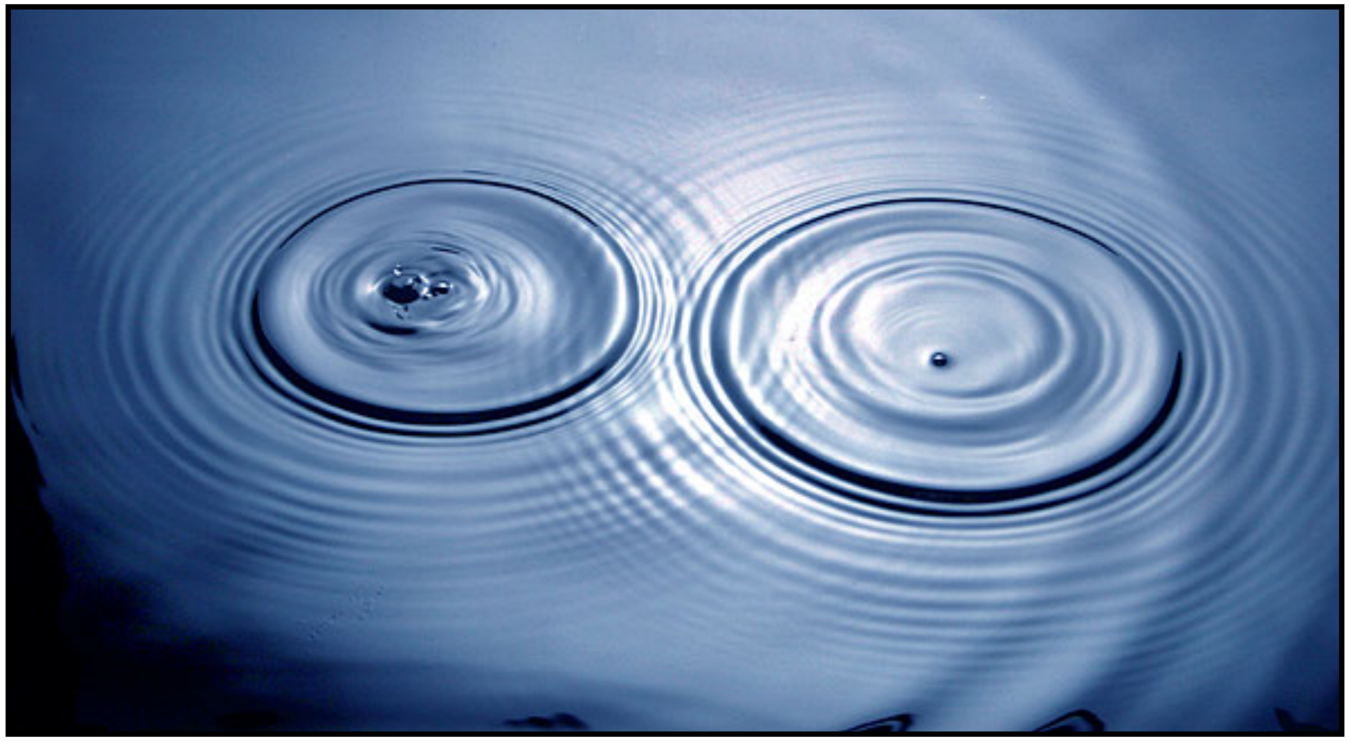
Interference between computational paths



The origin of quantum speedup

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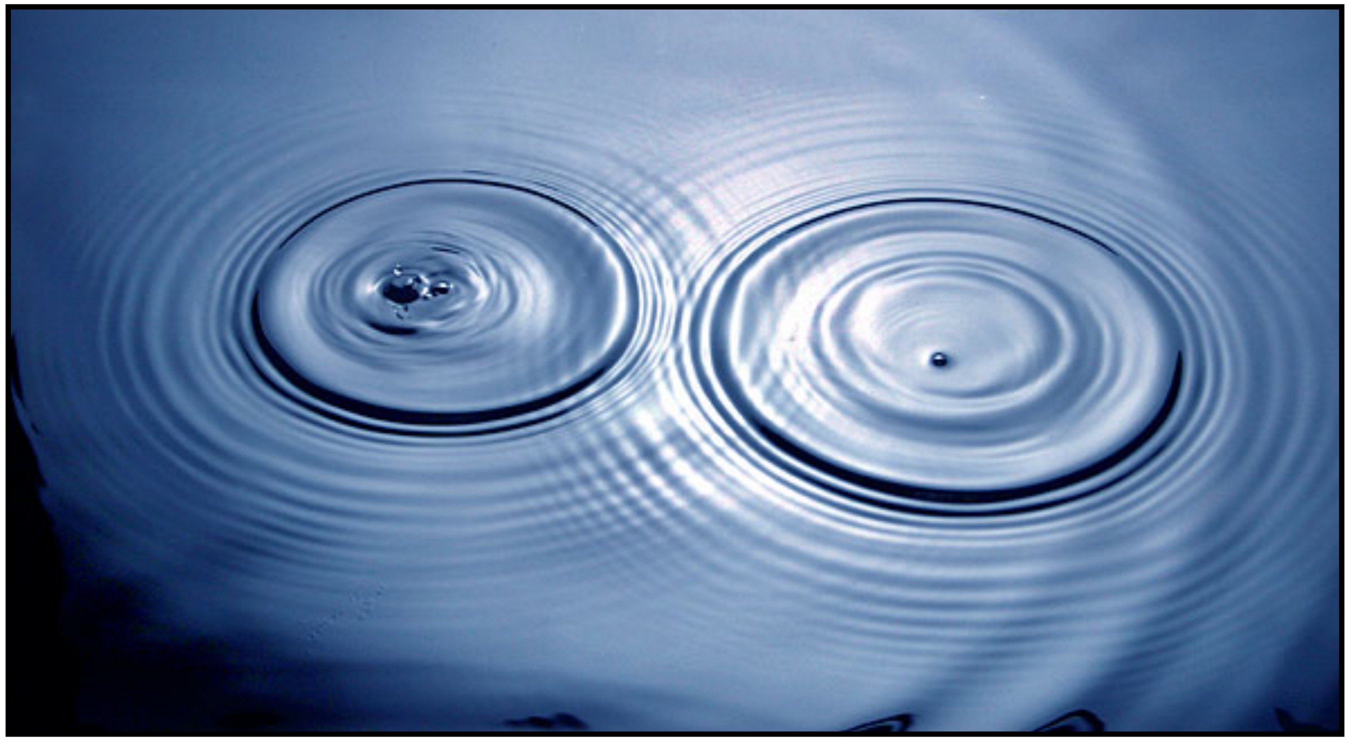

Arrange so that

- paths to the solution interfere constructively
- paths to non-solutions interfere destructively

The origin of quantum speedup

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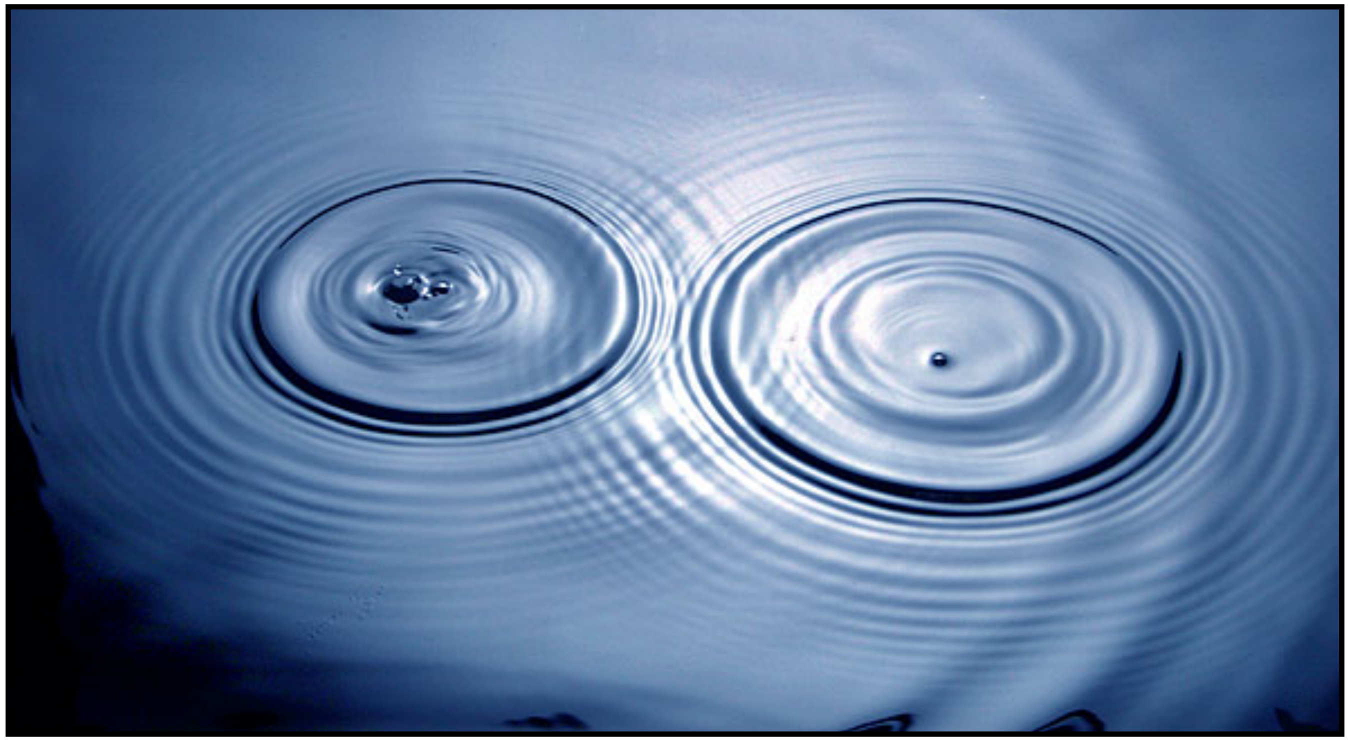

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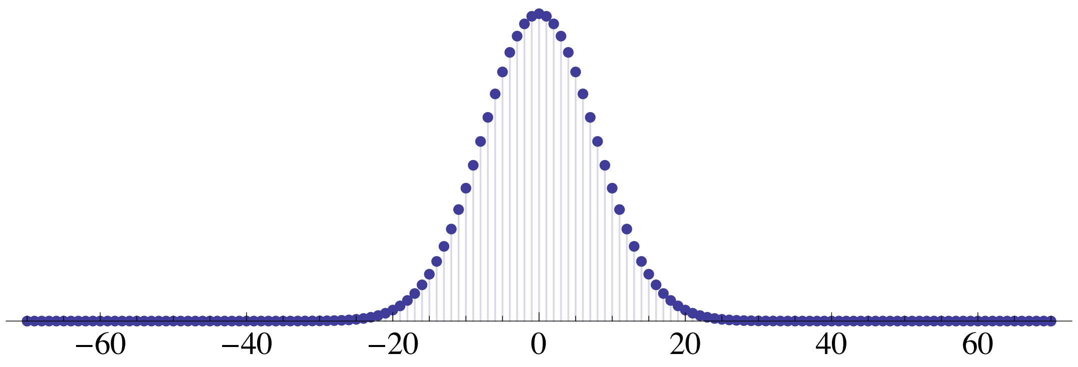
Quantum mechanics gives an efficient representation of complex interference phenomena

Quantum walk

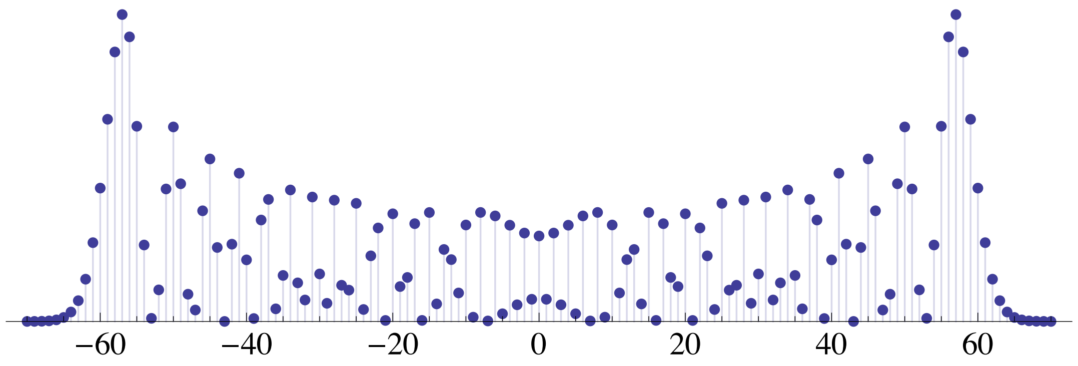
Quantum walk

Quantum analog of a random walk on a graph.

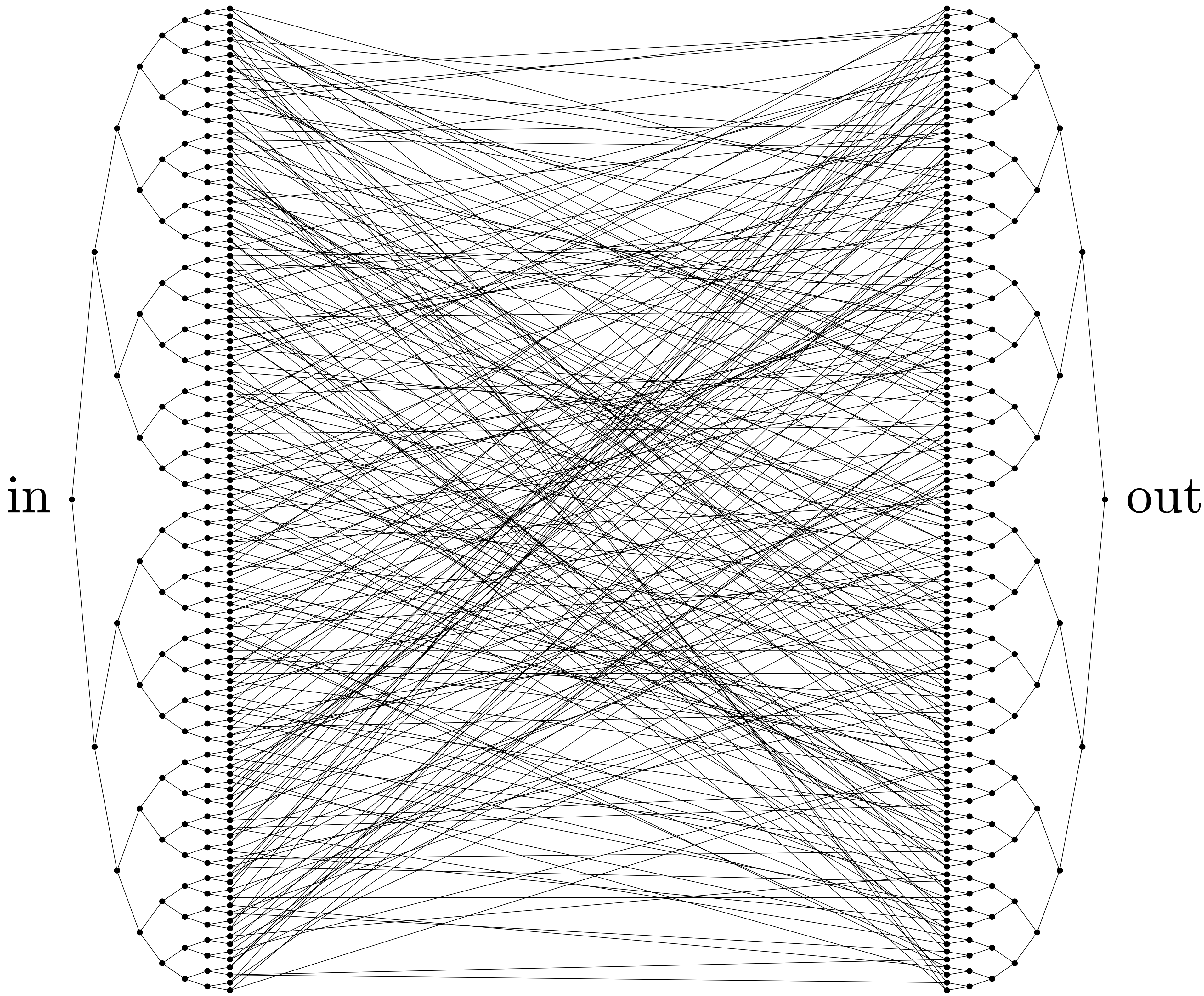
Idea: Replace probabilities by quantum amplitudes.
Interference can produce radically different behavior!



classical



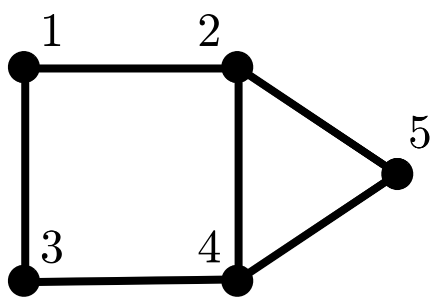
quantum



[Childs, Cleve, Deotto, Farhi, Gutmann, Spielman, STOC 2003]

From random walk to quantum walk

Graph G:



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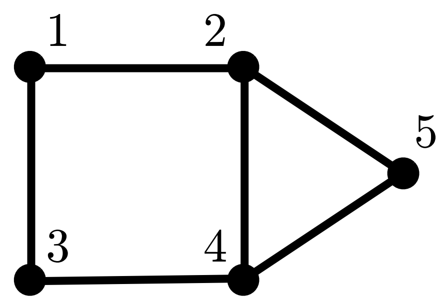

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

adjacency matrix

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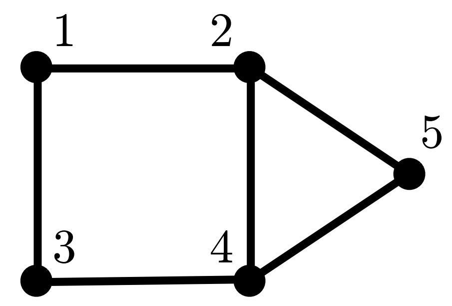
Graph G: 

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adjacency matrix *Laplacian*

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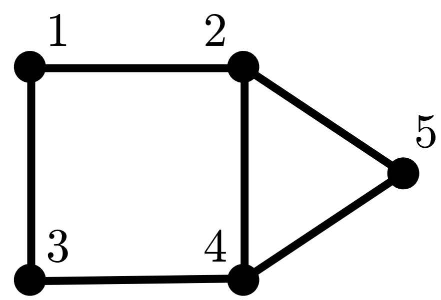

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Random walk on G

State: Probability $p_v(t)$ of being at vertex v at time t

Dynamics: $\frac{\mathrm{d}}{\mathrm{d}t}\vec{p} = L\vec{p}$

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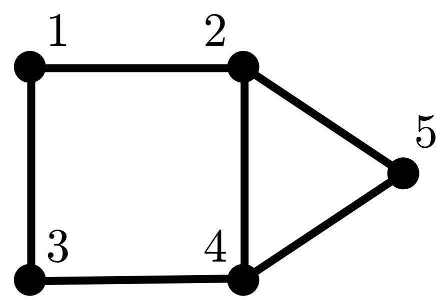

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State: Probability $p_v(t)$ of being at vertex v at time t

Dynamics: $$\frac{\mathrm{d}}{\mathrm{d}t}\vec{p} = L\vec{p}$$

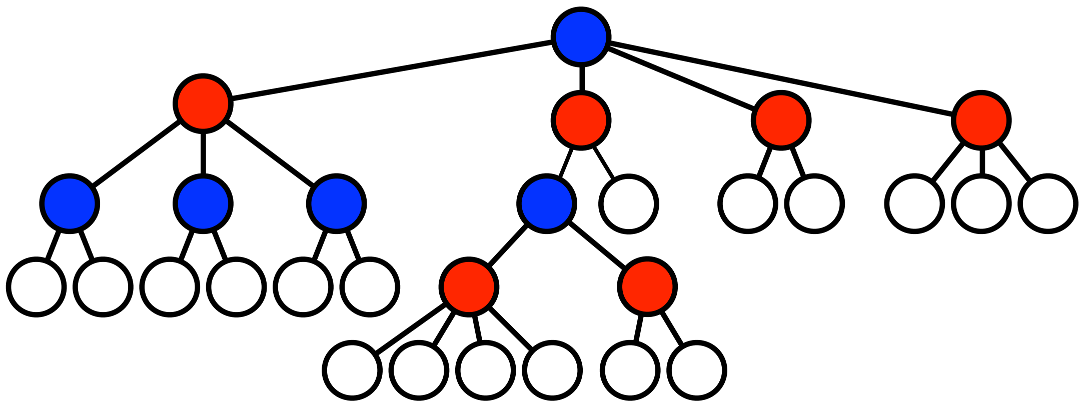
Quantum walk on G

State: Amplitude $a_v(t)$ to be at vertex v at time t

Dynamics: $$i\frac{\mathrm{d}}{\mathrm{d}t}\vec{a} = L\vec{a}$$

Outline

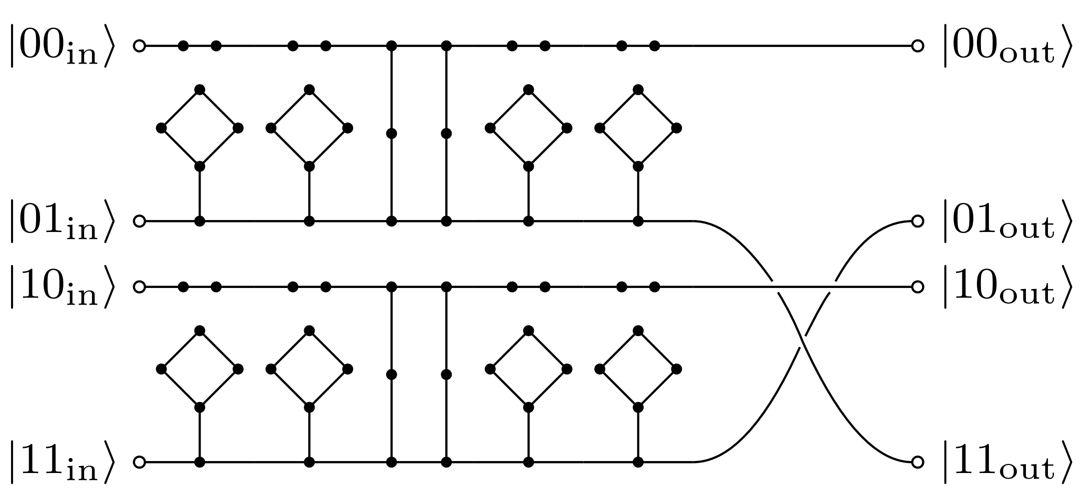
Formula evaluation



Quantum simulation

$$i \frac{\mathrm{d}}{\mathrm{d}t} \psi(t) = H\, \psi(t)$$

Universal computation

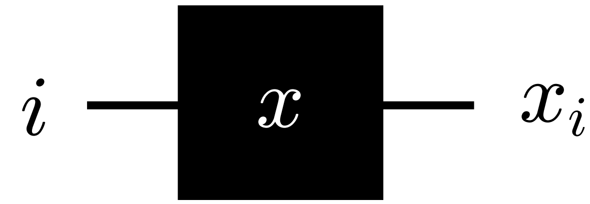


Formula evaluation

- Ambainis, Childs, Reichardt, Špalek, and Zhang, FOCS 2007, pp. 363–372; SIAM Journal on Computing **39**, 2513–2530 (2010)

Query complexity of formula evaluation

Query model: given a black box for a string $x \in \{0,1\}^n$



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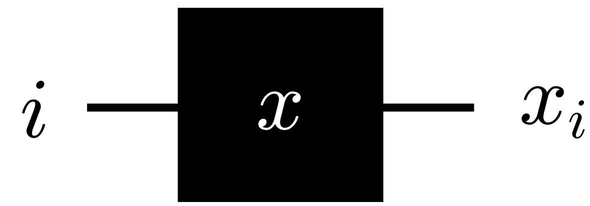

Compute some function of x using as few queries as possible

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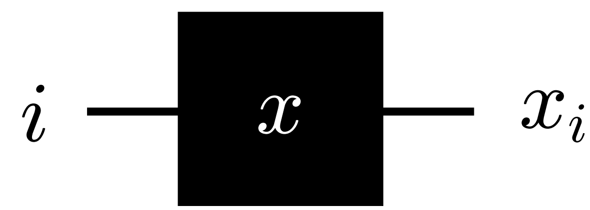

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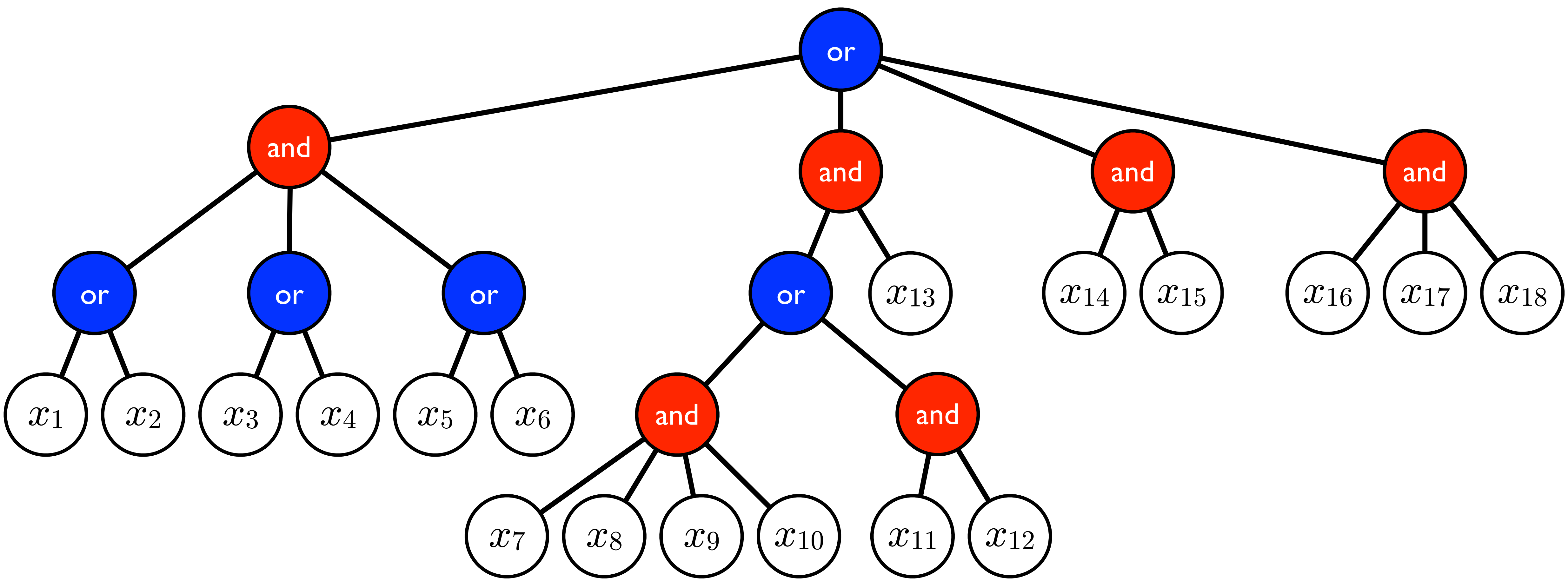
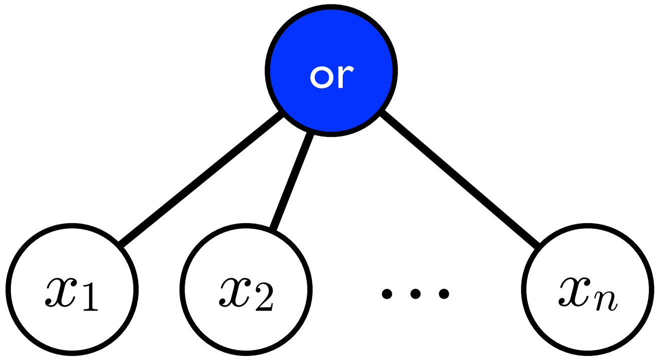


A single OR gate



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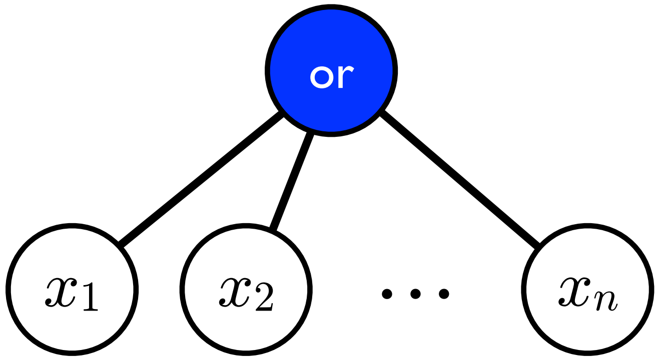

Classical complexity: $\Theta(n)$

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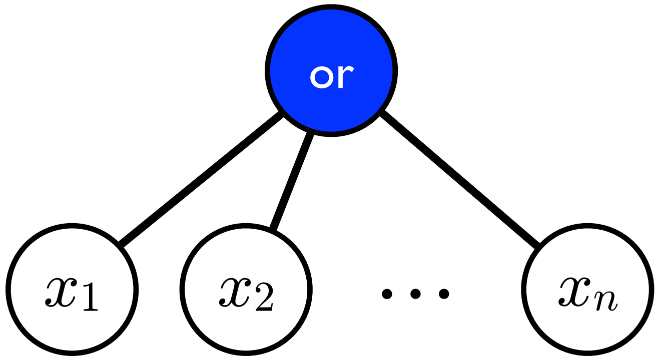

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Quantum algorithm [Grover 1996]: $O(\sqrt{n})$

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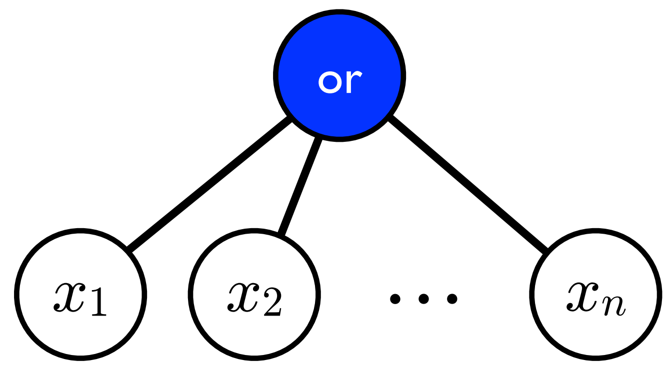
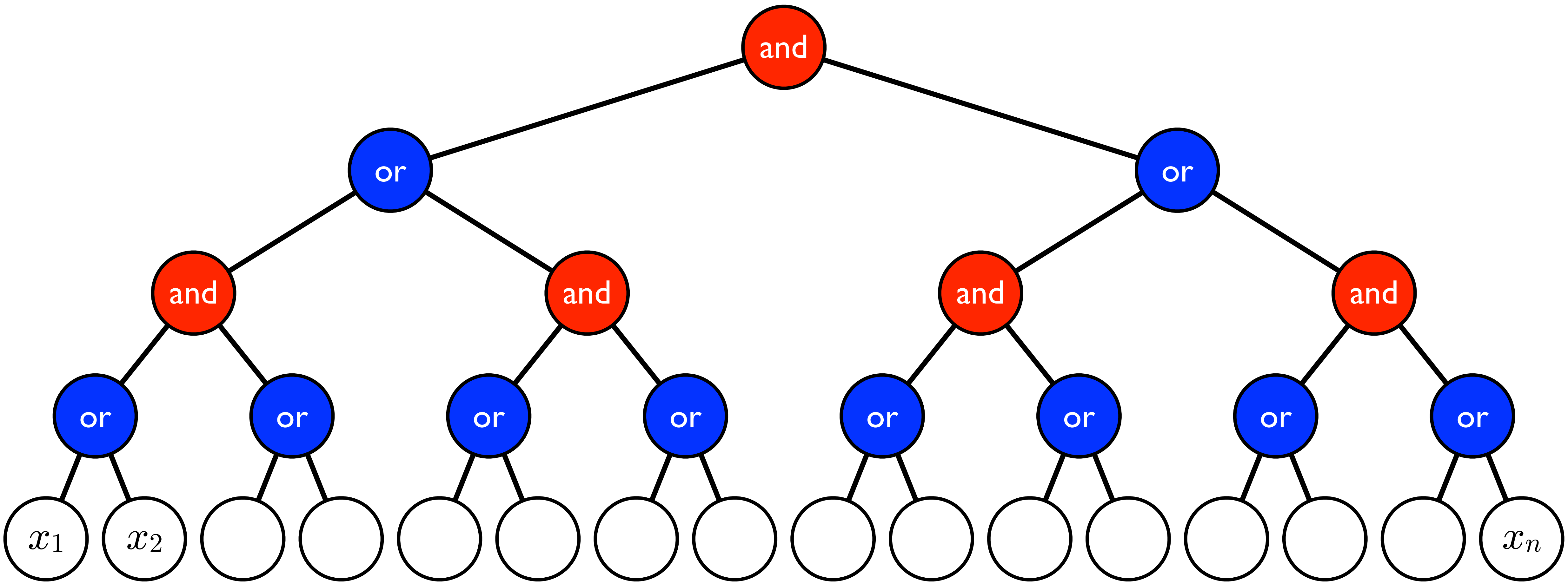


Classical complexity: $\Theta(n)$

Quantum algorithm [Grover 1996]: $O(\sqrt{n})$

Quantum lower bound [BBBV 1996]: $\Omega(\sqrt{n})$

Balanced binary AND-OR trees



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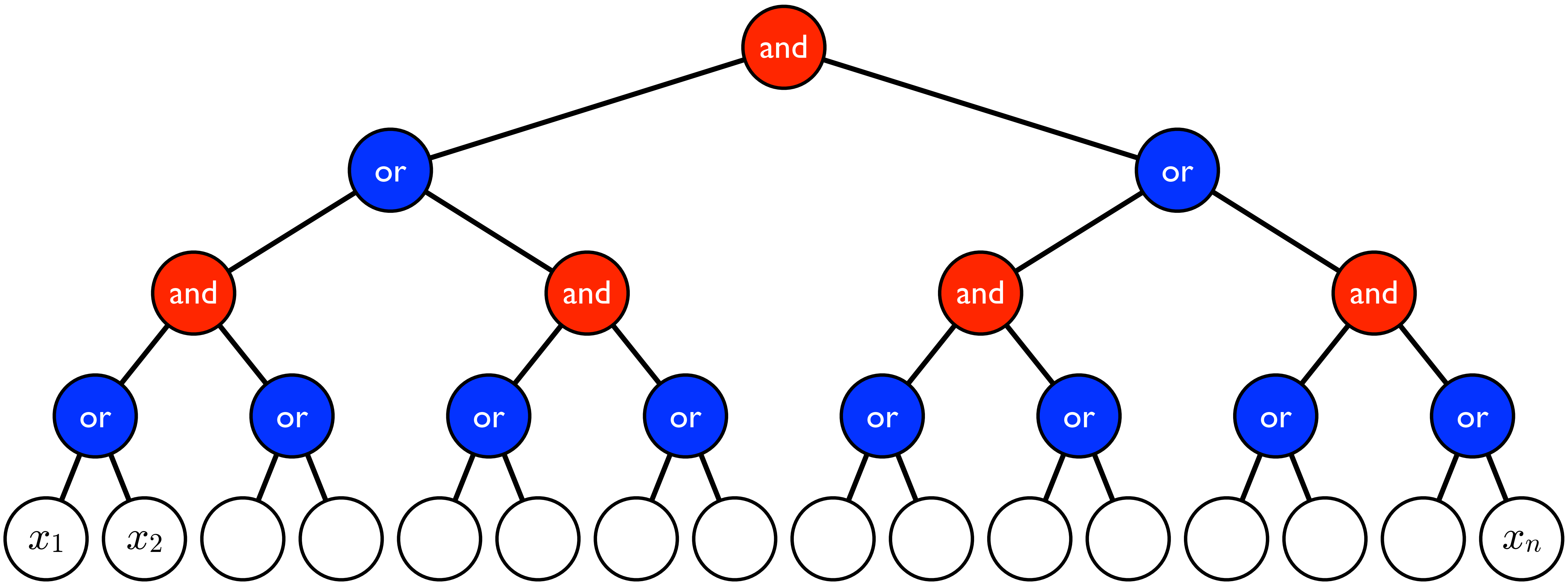

Classical complexity [Snir 85; Saks, Wigderson 86; Santha 95]: $\Theta(n^{0.753})$

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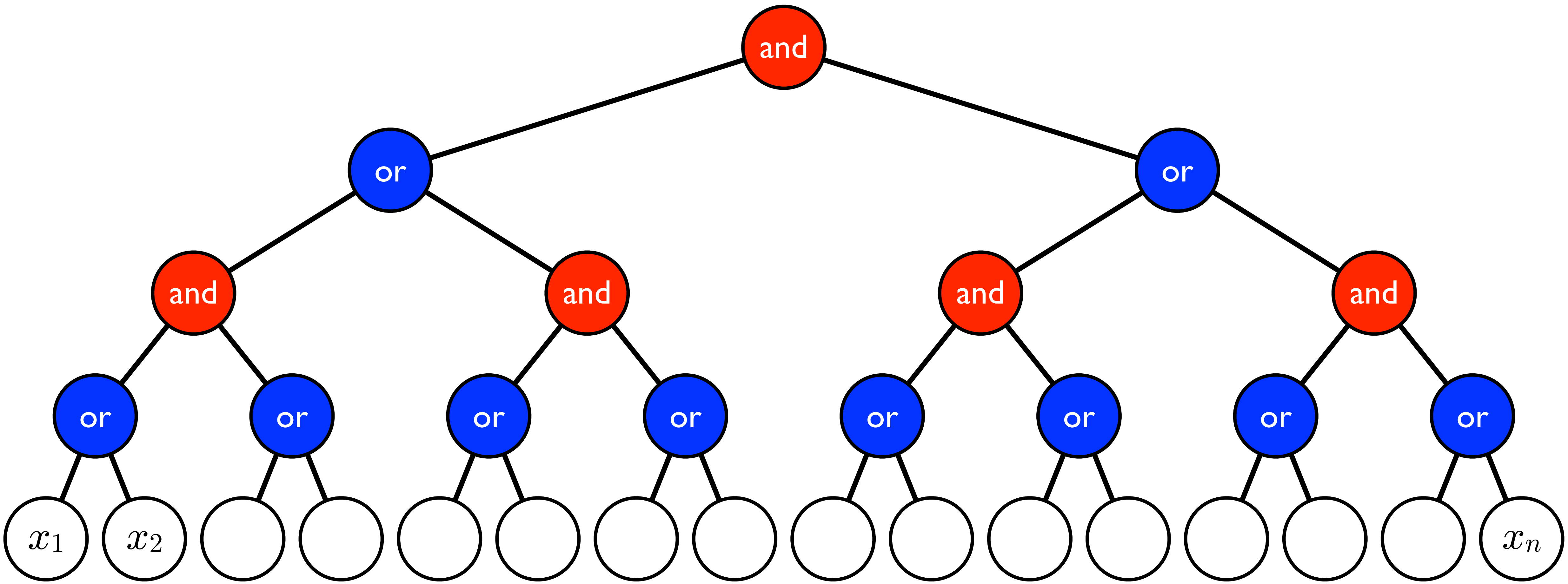

Classical complexity [Snir 85; Saks, Wigderson 86; Santha 95]: $\Theta(n^{0.753})$

Quantum lower bound [Barnum, Saks 02]: $\Omega(\sqrt{n})$
(holds for arbitrary AND-OR formulas)

Formula evaluation by scattering

[Farhi, Goldstone, Gutmann 07]

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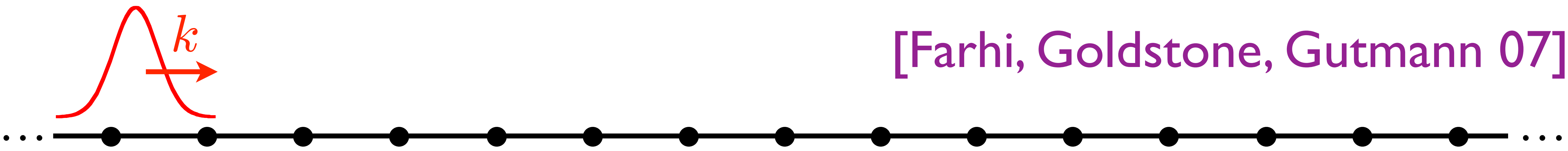

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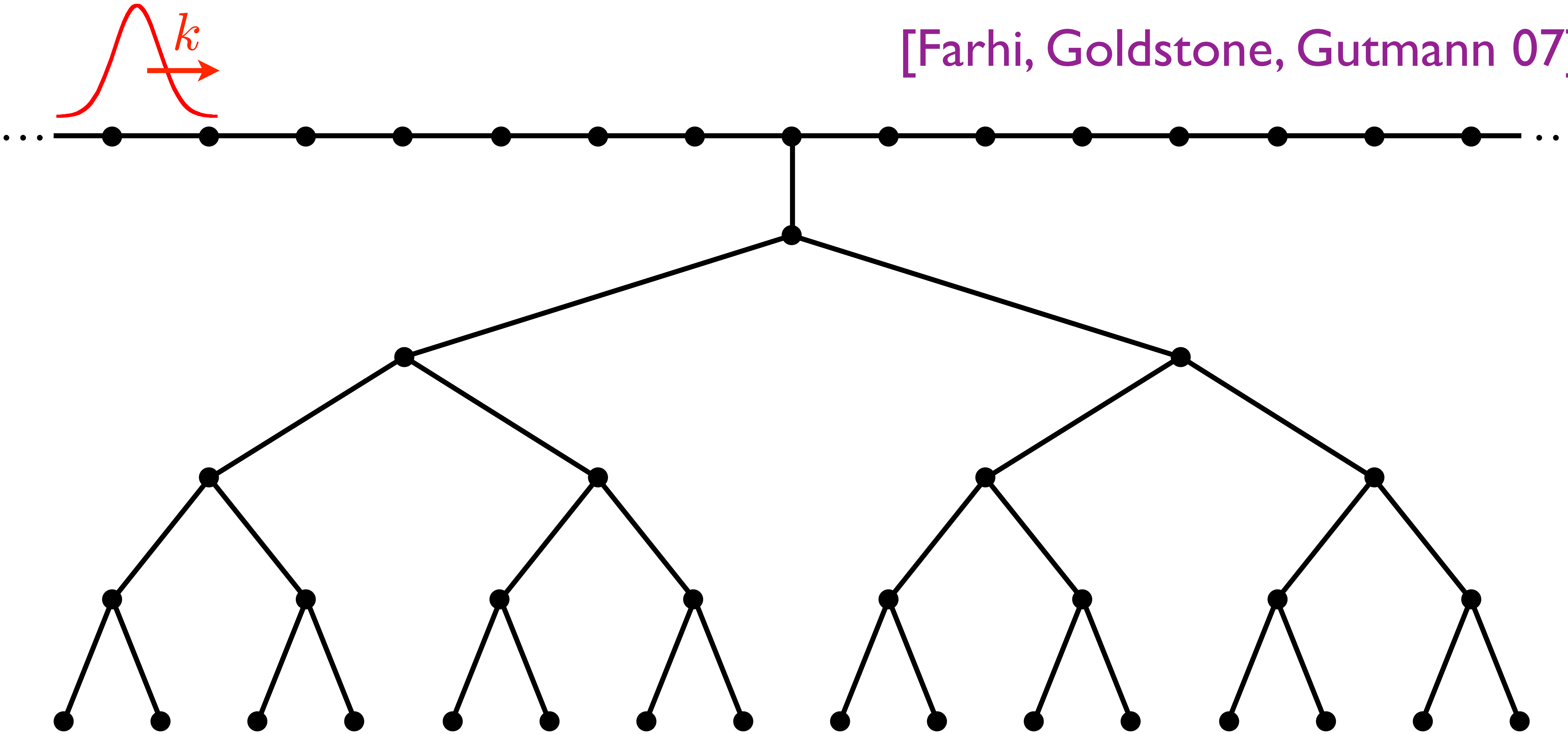

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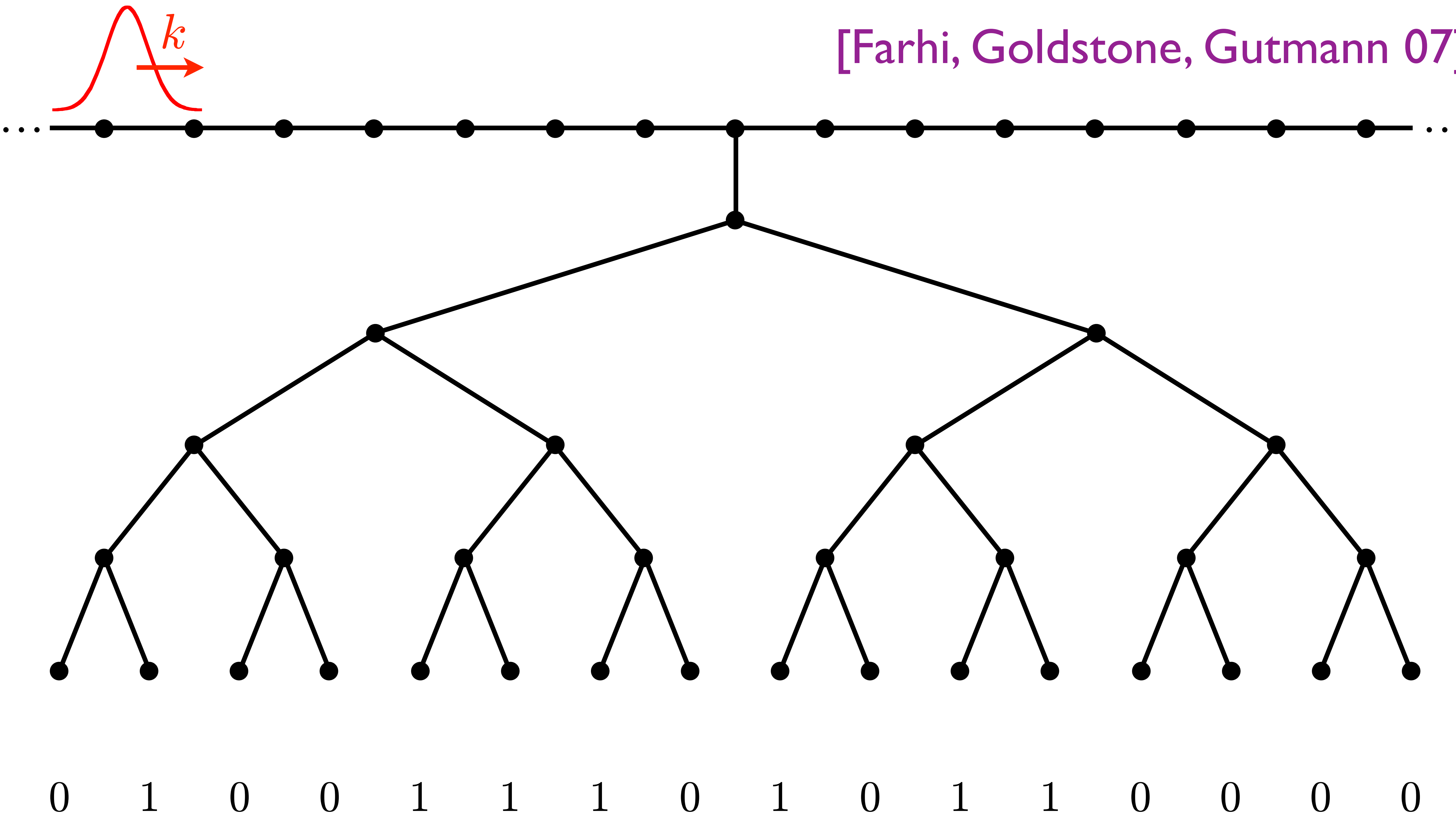

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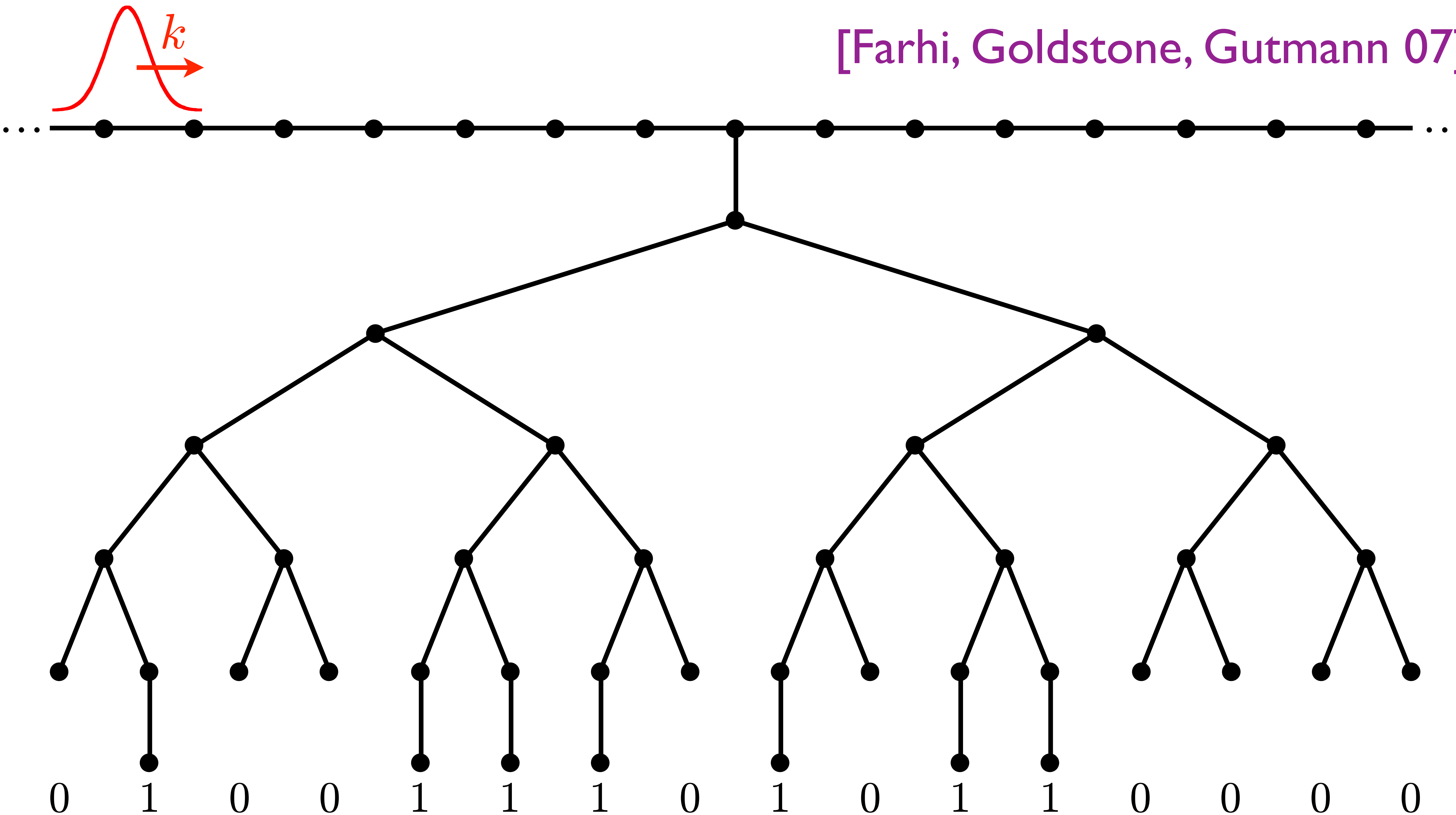

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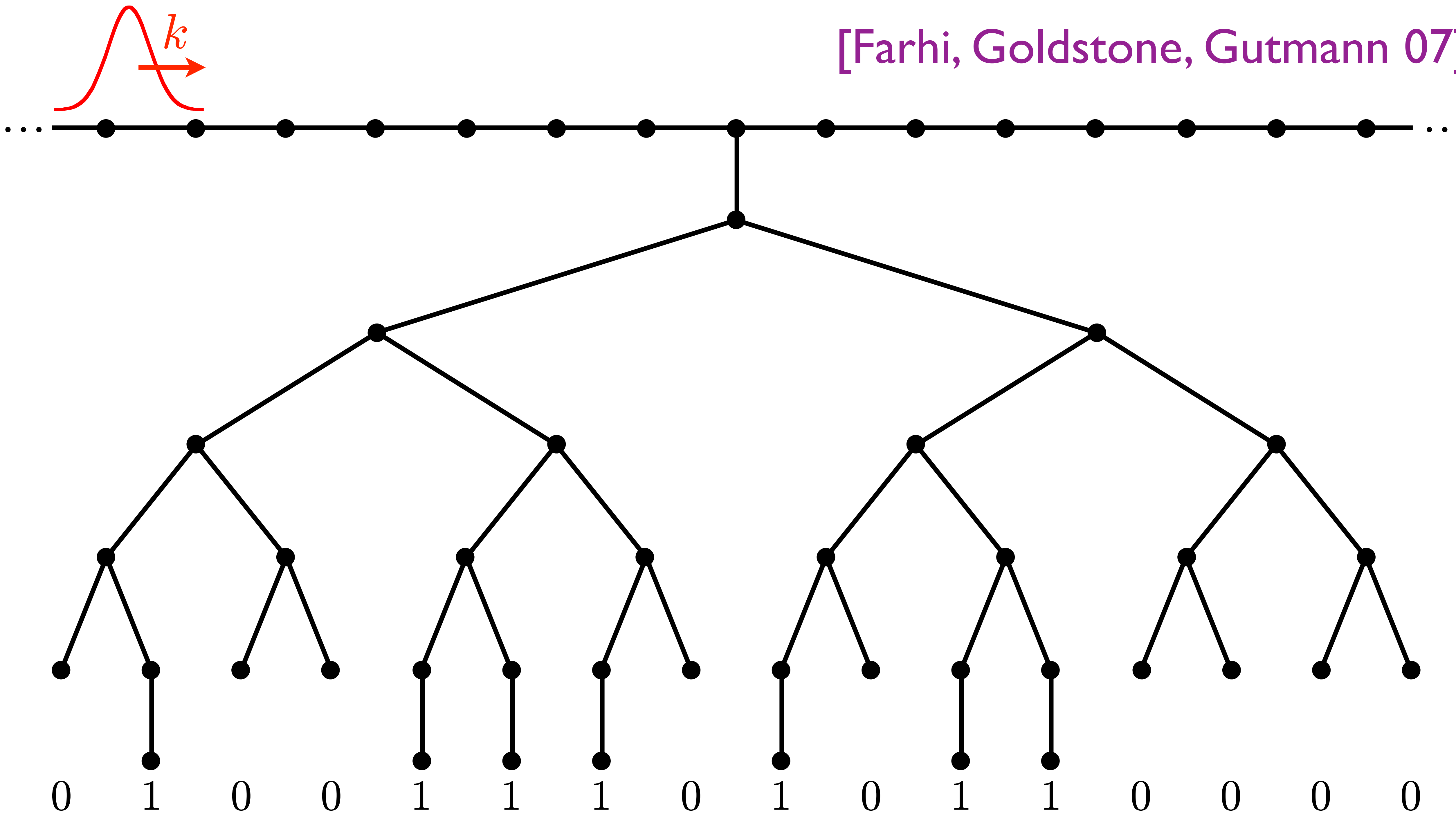

Claim: For small k, the wave is transmitted if the formula (translated into NAND gates) evaluates to 0, and reflected if it evaluates to 1.

General formulas

This simple strategy does not work for general formulas.

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To get a general algorithm:

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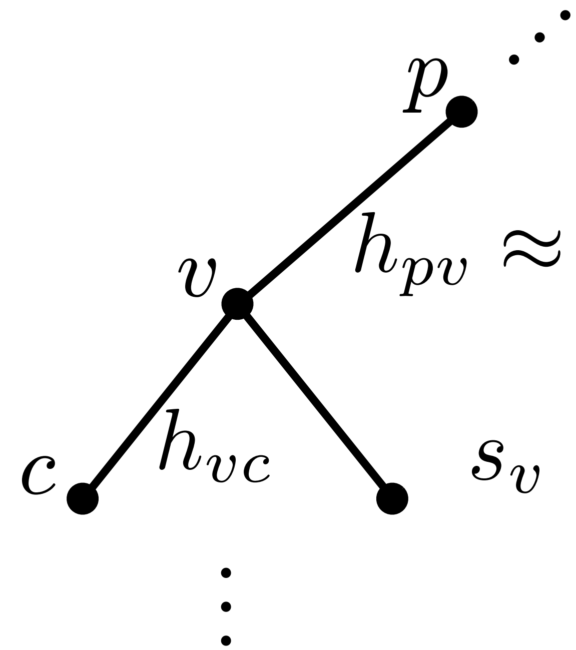

$$h_{pv} \approx \sqrt[4]{\frac{s_v}{s_p}}$$

$s_v =$ # of inputs in subformula under v

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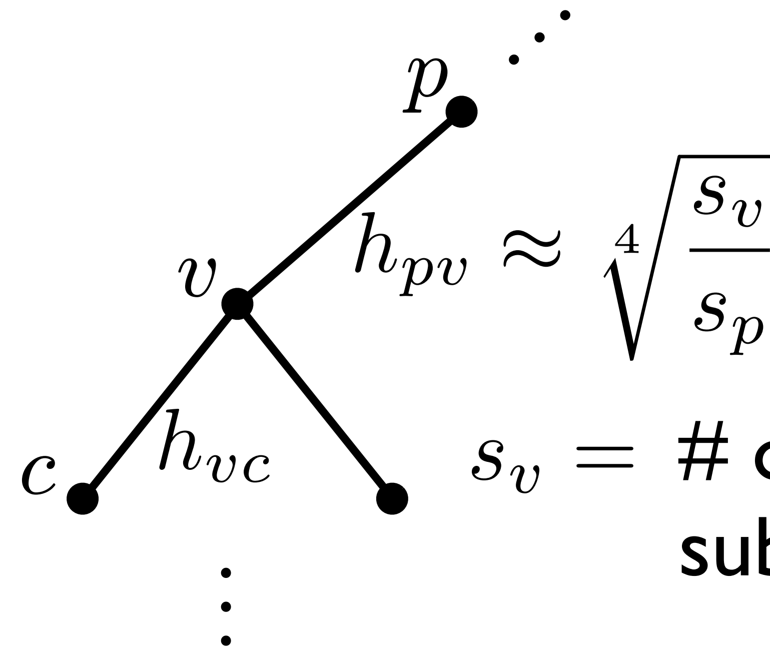

$s_v =$ # of inputs in subformula under v

Lemma: If the formula evaluates to 0, then the tree has an eigenstate with eigenvalue 0 that has constant overlap on the root. If the formula evaluates to 1, then all eigenstates with eigenvalue $O(1/\sqrt{n})$ have no overlap on the root.

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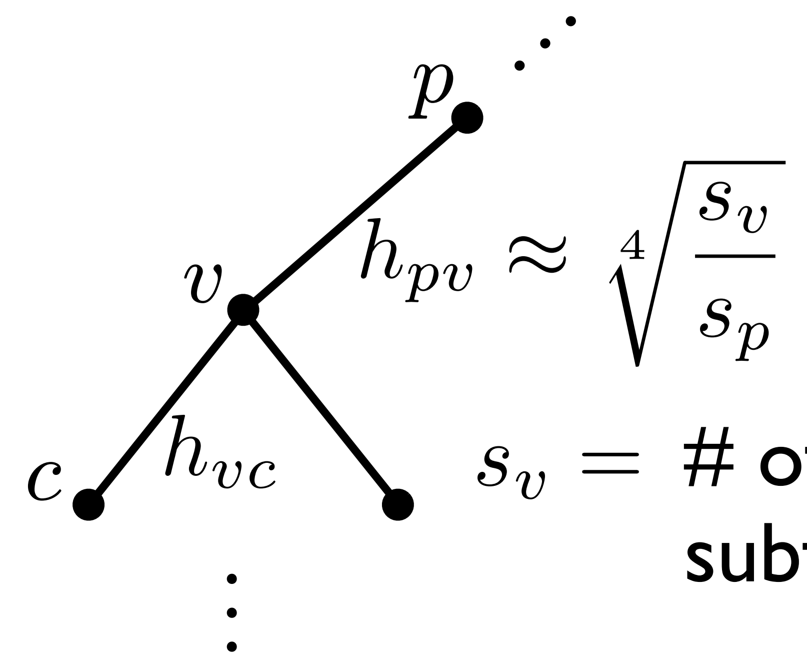

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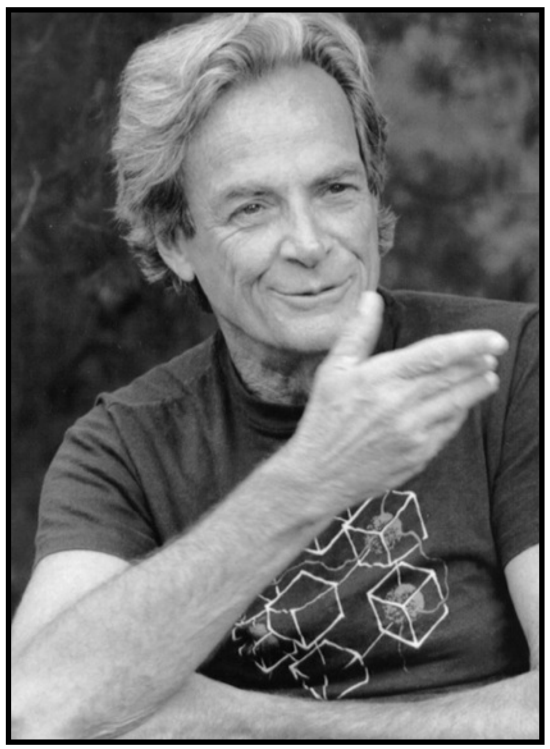
> **Lemma:** If the formula evaluates to 0, then the tree has an eigenstate with eigenvalue 0 that has constant overlap on the root. If the formula evaluates to 1, then all eigenstates with eigenvalue $O(1/\sqrt{n})$ have no overlap on the root.

The quantum query complexity of evaluating any AND-OR formula is $O(n^{\frac{1}{2}+\epsilon})$ (subsequently improved to $O(\sqrt{n})$ [Reichardt 10])

Quantum simulation

- Childs, Communications in Mathematical Physics **294**, 581–603 (2010)
- Berry and Childs, Quantum Information and Computation **12**, 29–62 (2012)



"... nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

Richard Feynman
Simulating physics with computers (1981)

Why simulate quantum mechanics?

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Computational chemistry/physics

- chemical reactions
- properties of materials

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Implementation of quantum algorithms

- continuous-time quantum walk
- adiabatic quantum computation
- linear equations

Quantum dynamics

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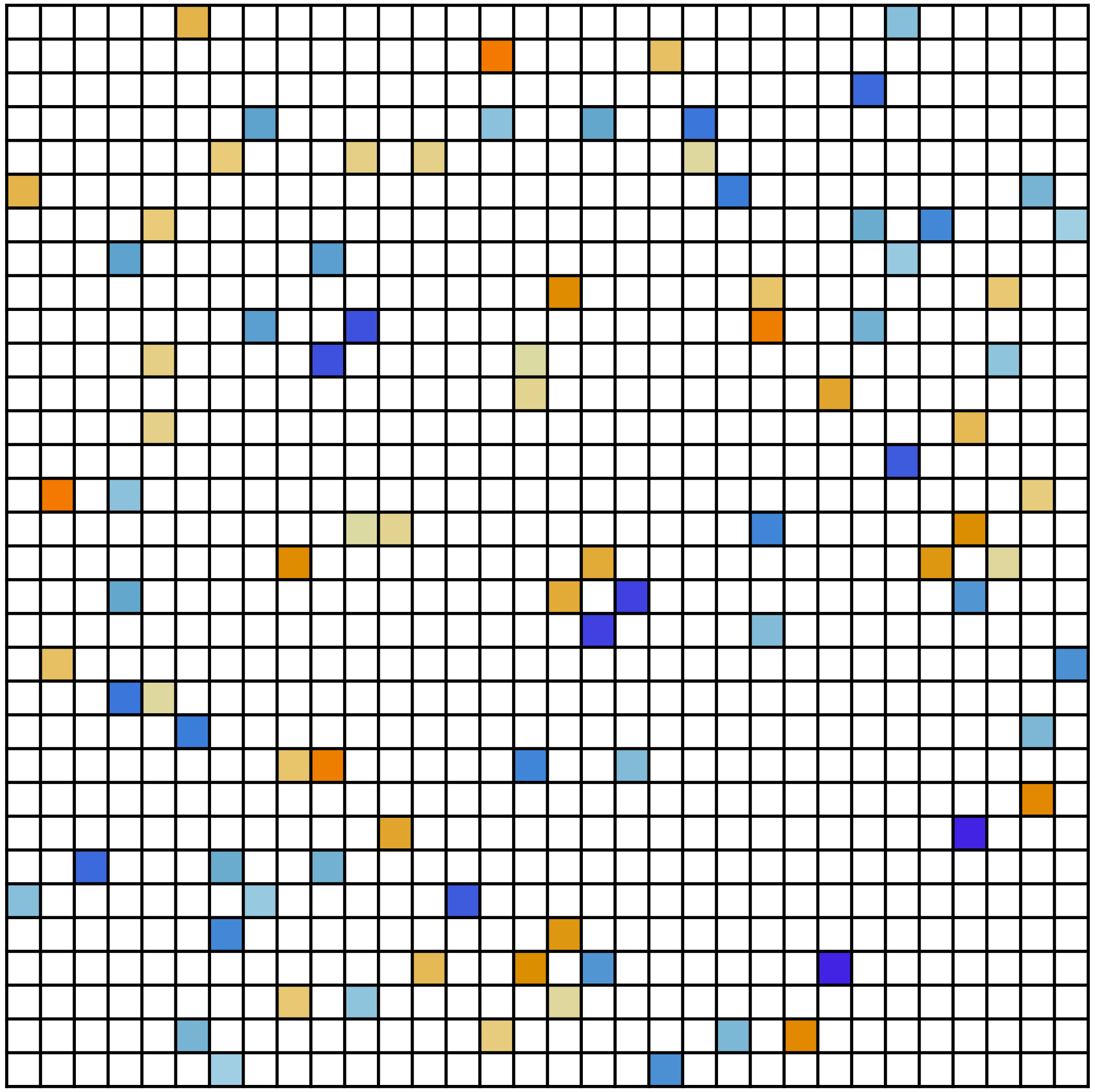
By performing measurements on the final state, a quantum computer can efficiently answer questions that (apparently) a classical computer cannot

Sparse Hamiltonians

At most d nonzero entries per row (here $d = 4$)

$$H = $$



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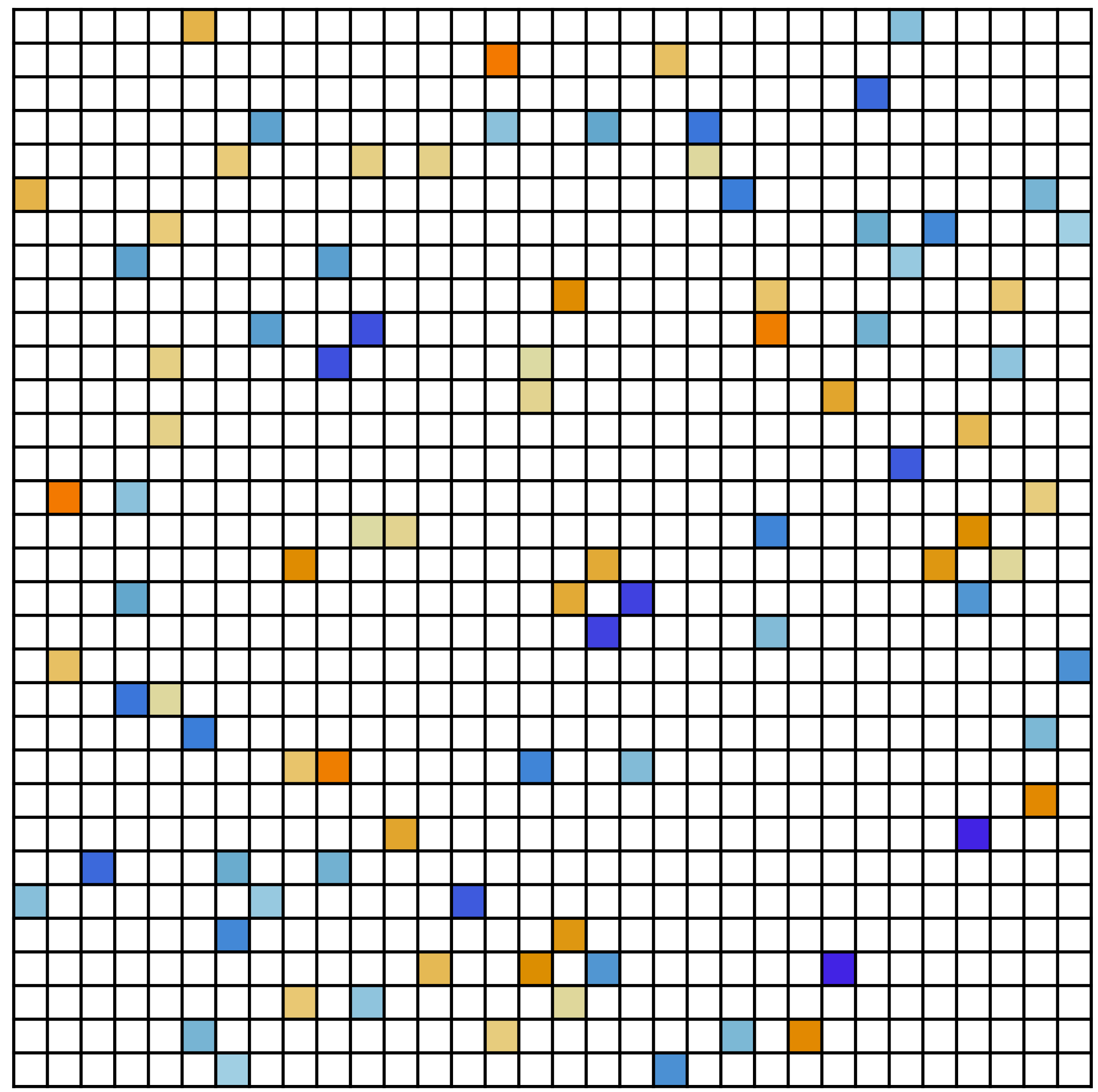
$H =$ 

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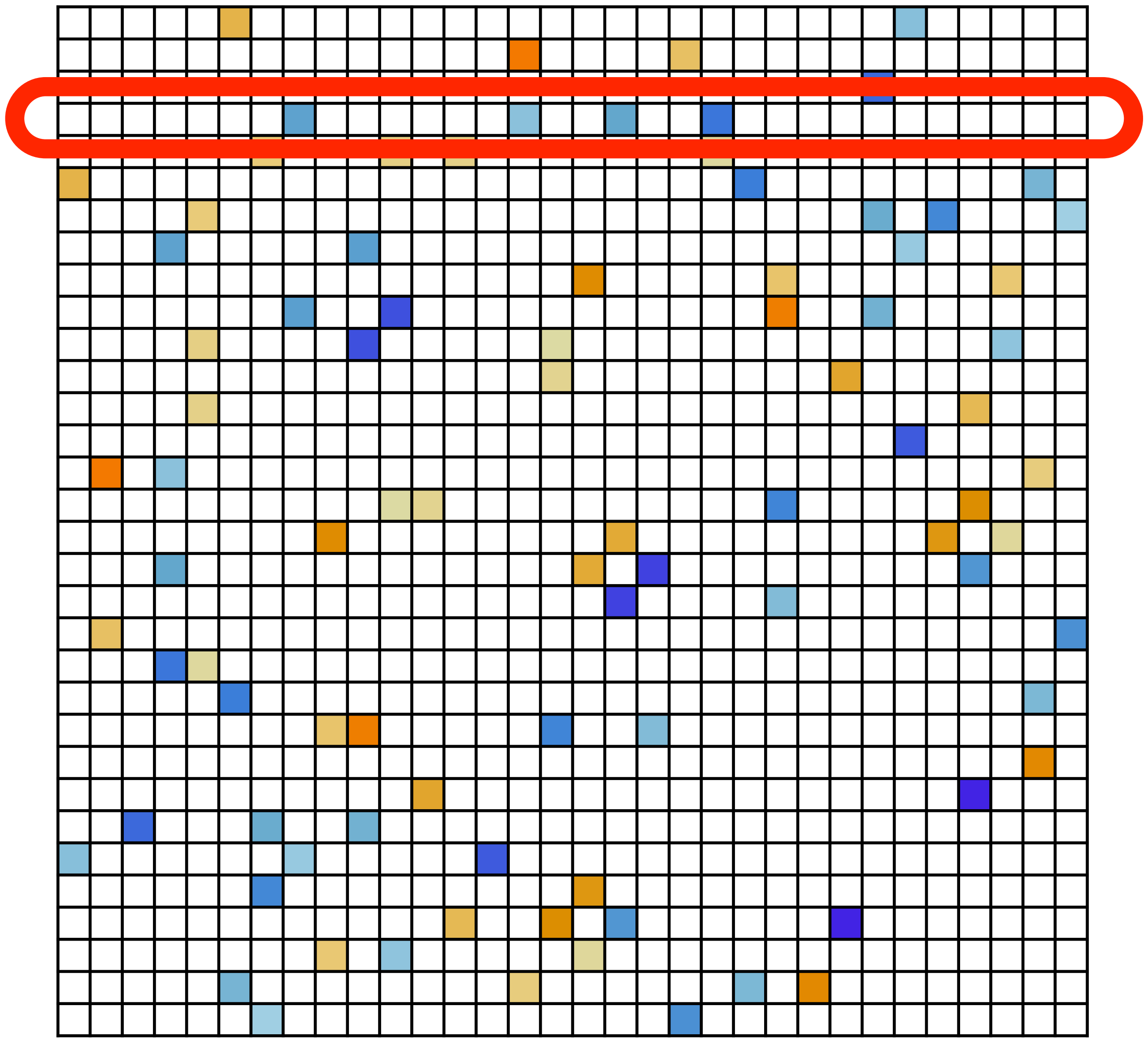
$H =$ 

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Simulation via product formulas

Original approach to sparse Hamiltonian simulation:

- Decompose $H = \sum_j H_j$ where each H_j is 1-sparse
  (distributed edge coloring)
- Recombine terms
  (product formulas, e.g., $e^{-i(A+B)t} \approx (e^{-iAt/r} e^{-iBt/r})^r$)

[AT 03, CCDFGS 03, BACS 07, CK 10]

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Running time of the best approach of this kind:

- Superlinear in evolution time t
- Cubic in sparsity d

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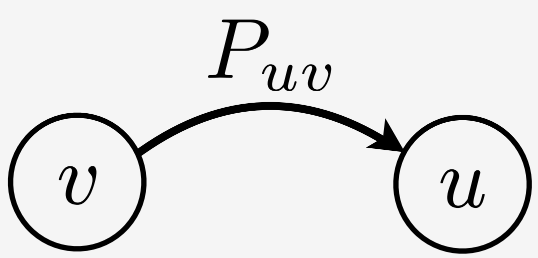

- Reflect about $\mathrm{span}\{\psi_v : v \in V\}$

  where $(\psi_v)_{(w,u)} = \begin{cases} \sqrt{P_{uv}} & \text{if } v = w \\ 0 & \text{otherwise} \end{cases}$

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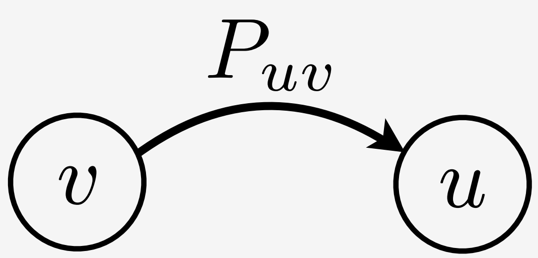

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This gives a quantum analog of the Markov chain P

Hamiltonian simulation by quantum walk

1. Define an analog of Szegedy's walk for any Hamiltonian H (in place of the stochastic matrix P)
2. Show how to perform steps of this walk using queries to the sparse Hamiltonian
3. Relate the spectrum of the walk to the spectrum of H
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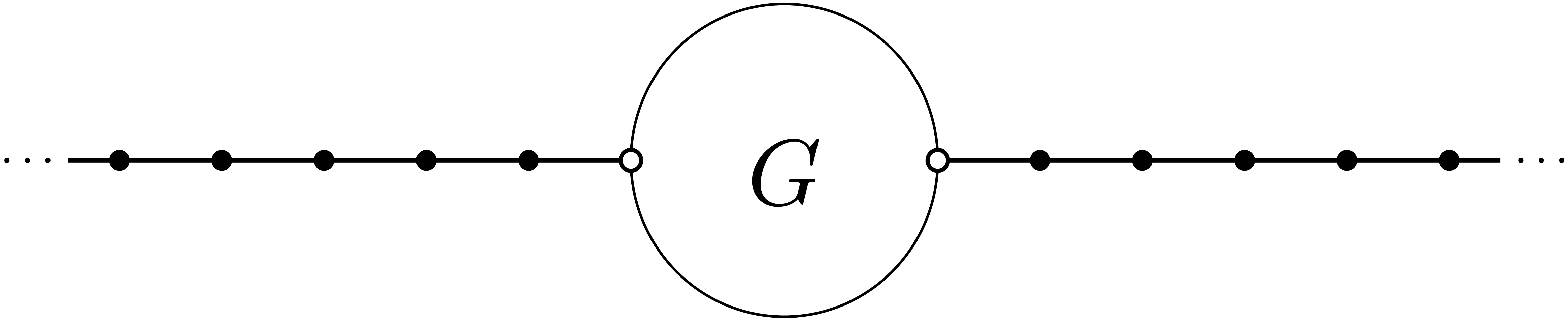
This algorithm is optimal with respect to either d or t alone

Universal computation

- Childs, Physical Review Letters **102**, 180501 (2009)
- Childs, Gosset, and Webb, Science **339**, 791–794 (2013)

Scattering on graphs

Attach semi-infinite paths to two vertices of an arbitrary finite graph.



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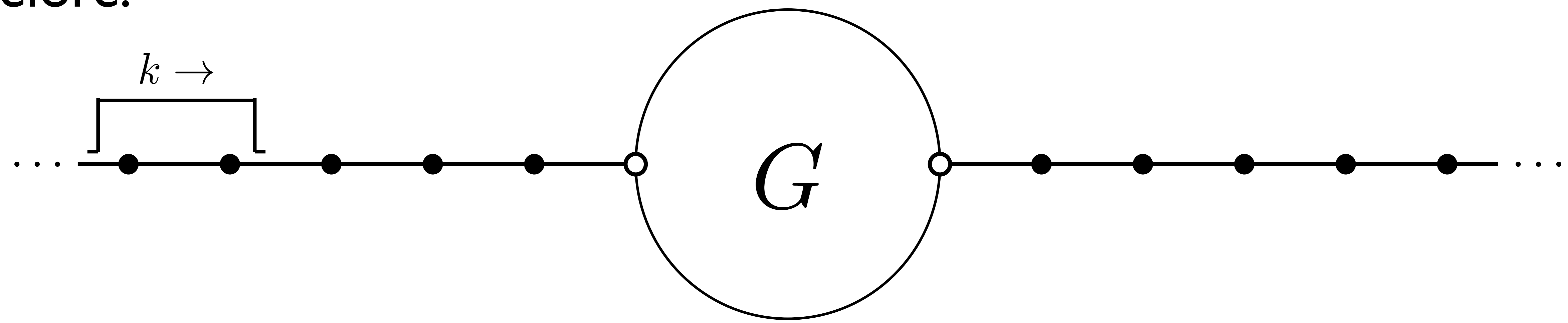

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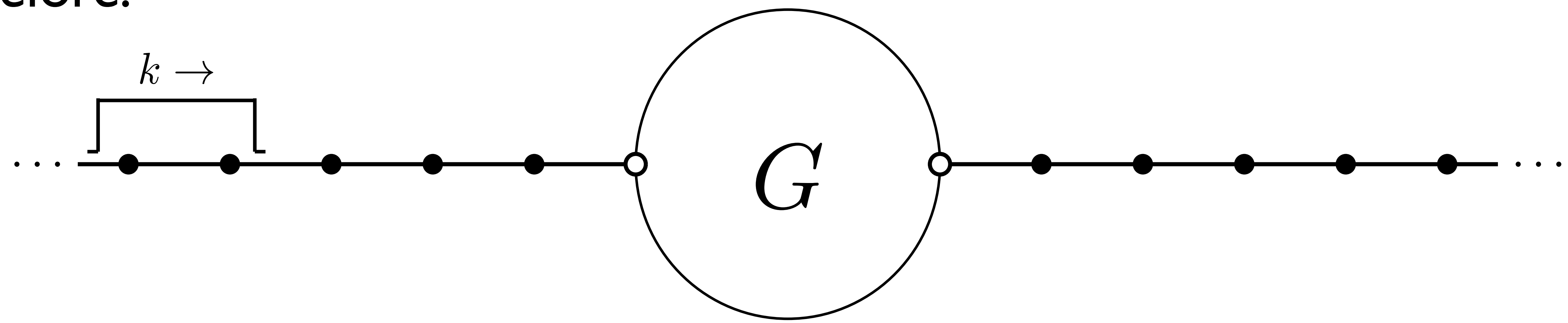

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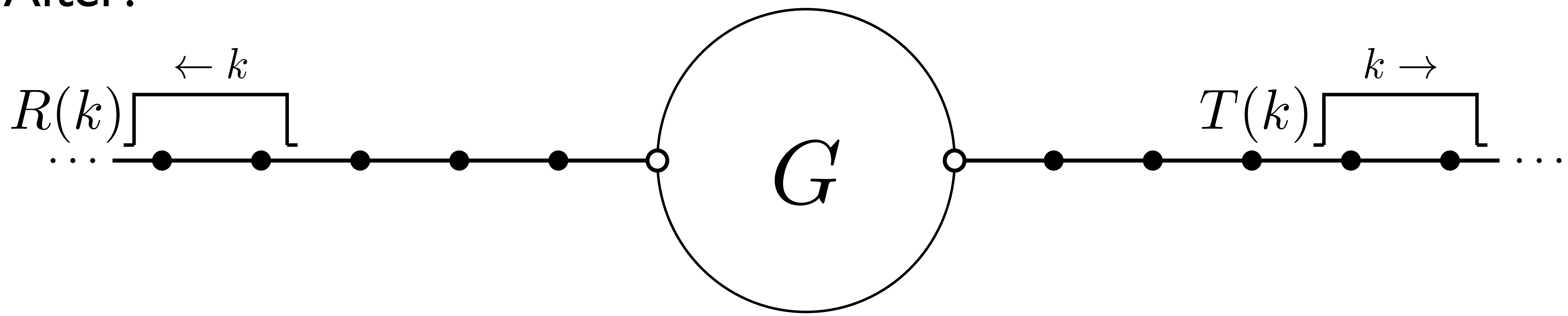

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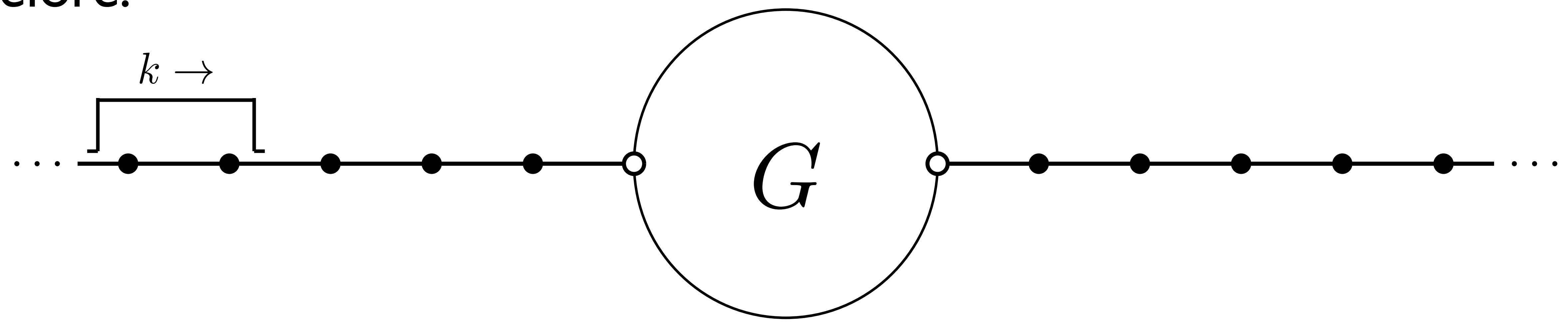

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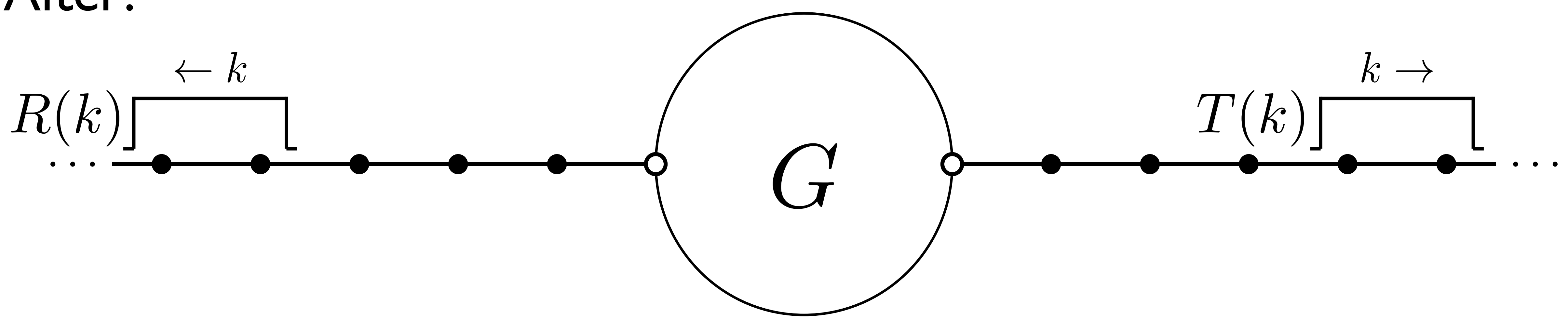
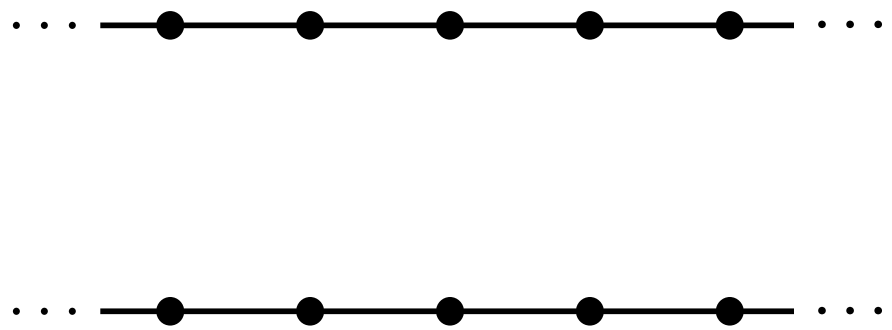


More generally, attach any number of semi-infinite paths. The scattering behavior is described a unitary matrix called the *S-matrix*.

Implementing a gate



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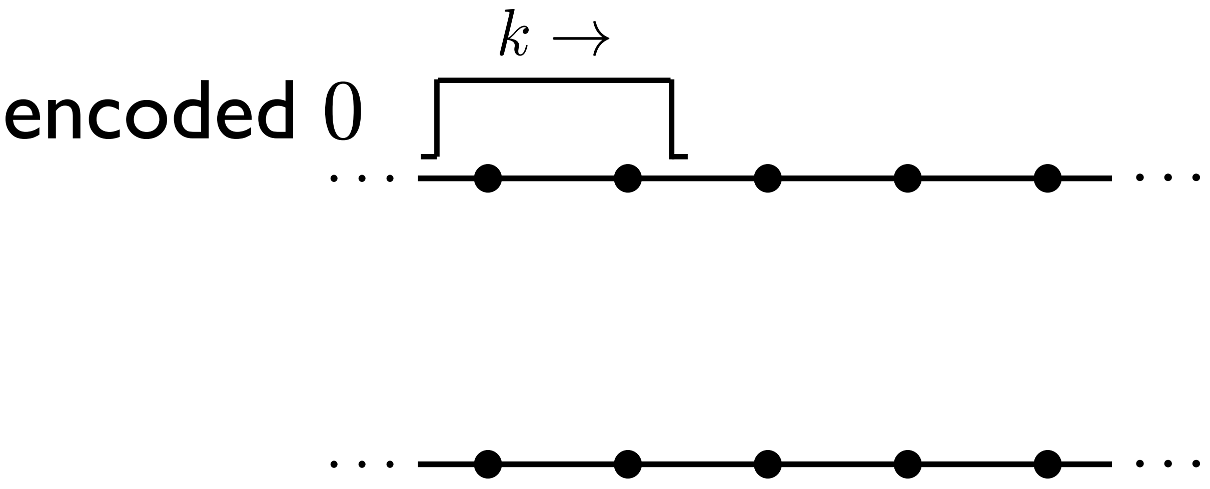

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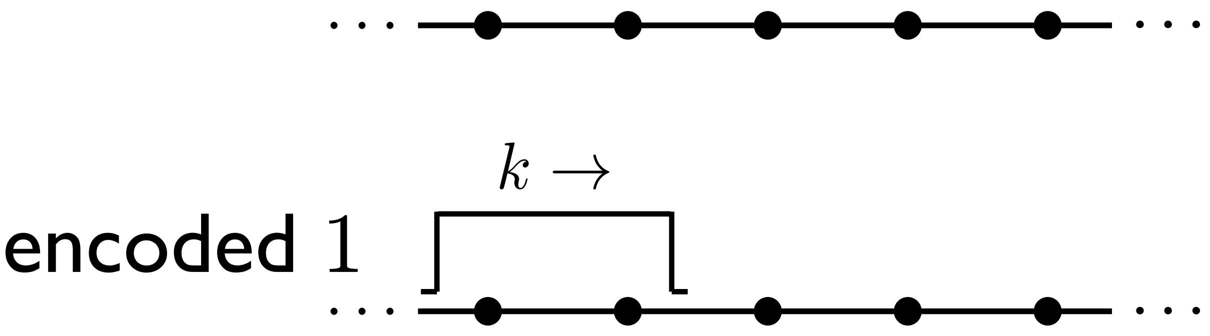

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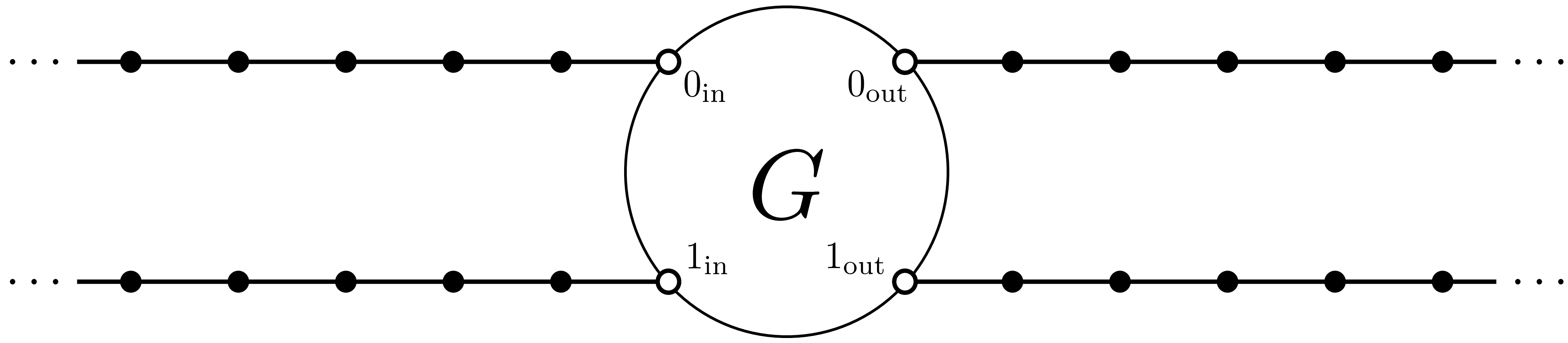

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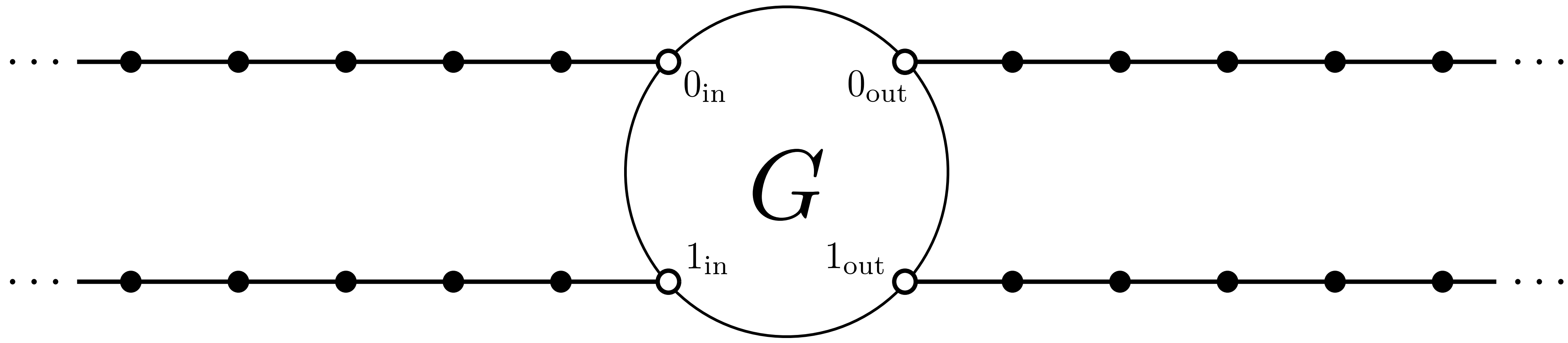

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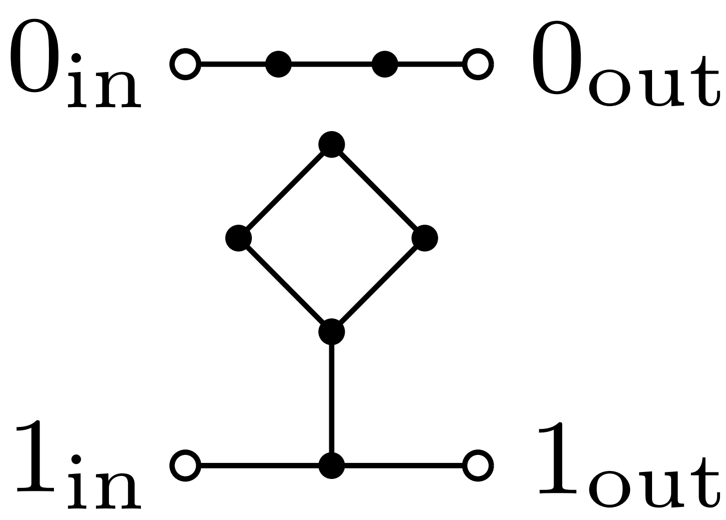
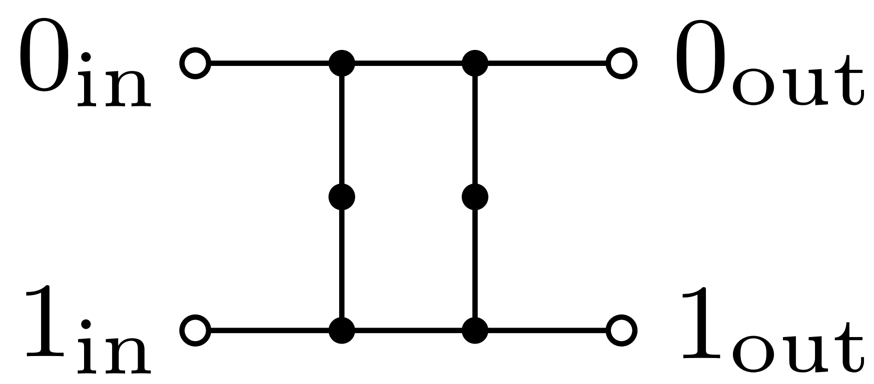
$$S(k) = \begin{pmatrix} 0 & V \\ U & 0 \end{pmatrix}$$

Universal set of single-qubit gates





$$\begin{pmatrix} 1 & 0 \\ 0 & \sqrt{i} \end{pmatrix}$$

$$-\frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$$

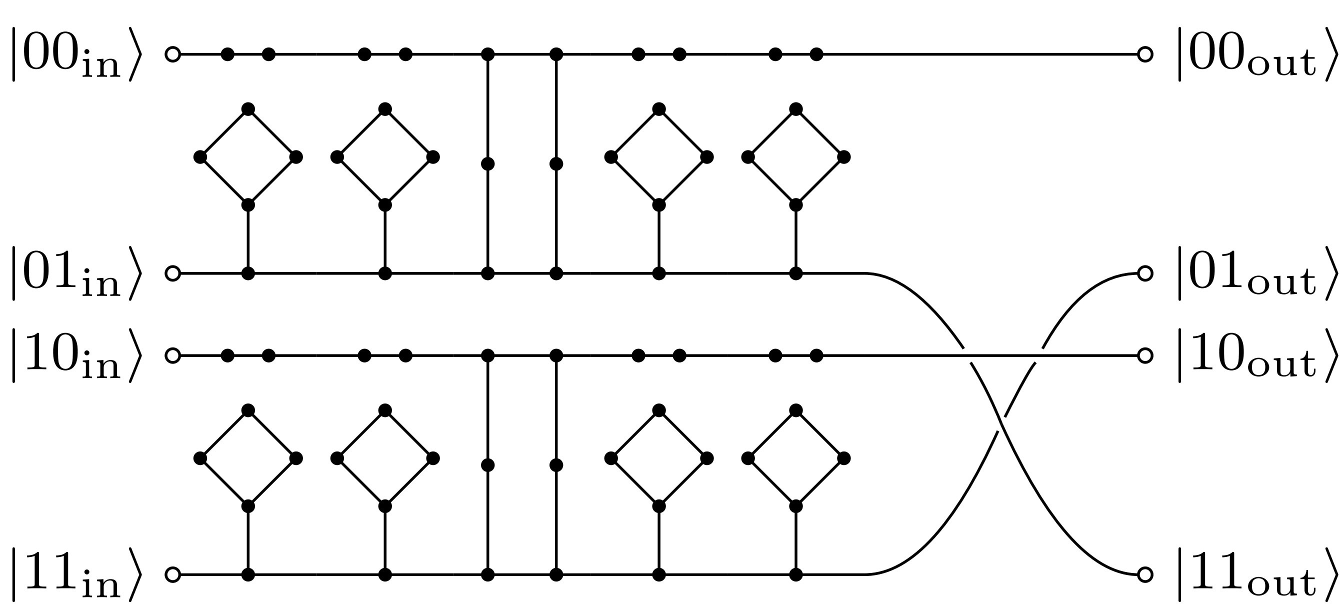
momentum for logical states: $k = \pi/4$

Universality construction

With an appropriate encoding of n-qubit states, two-qubit gates are trivial.

Implement sequences of gates by concatenation.

Result: Any n-qubit circuit can be simulated by some graph.



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… but not a new architecture (the graph is necessarily exponentially large).

Quantum walks with many walkers

Consider a quantum walk with many walkers that interact locally

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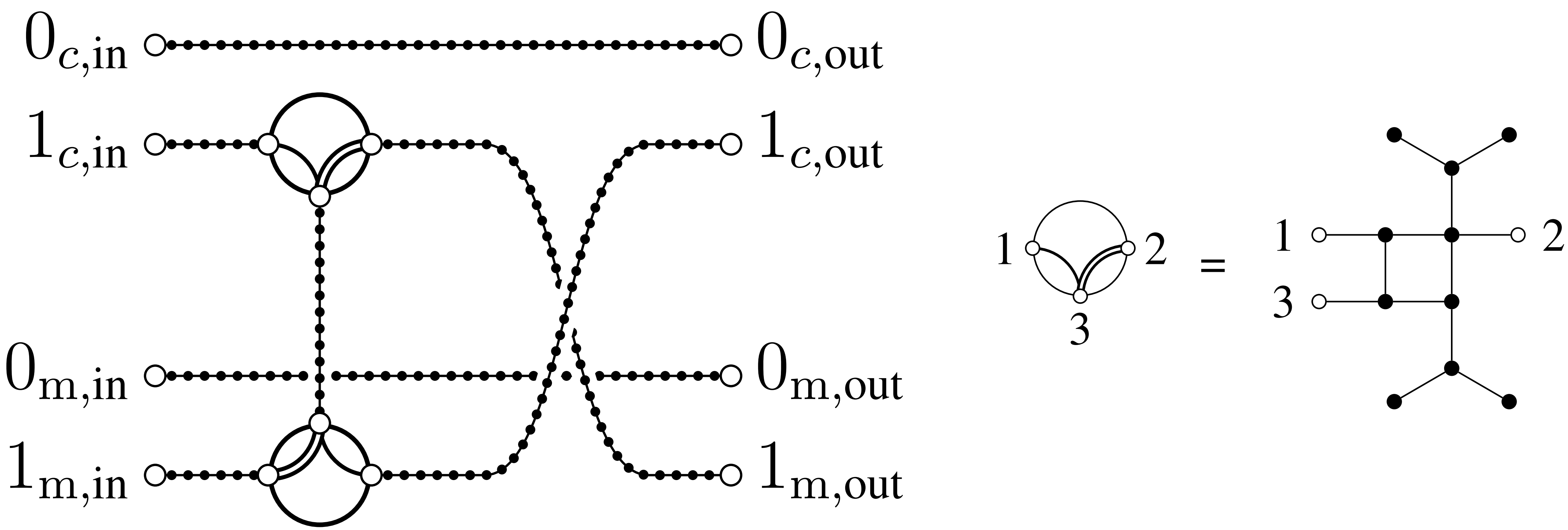
- Quantum walks with many interacting walkers (on small graphs) are also computationally powerful
- New architecture for a quantum computer (with no time-dependent control)
- Simulating the dynamics of interacting many-body systems is BQP-hard (e.g., the "Bose-Hubbard model" on a sparse, unweighted, planar graph)

Universal computation with many walkers

Main new idea: a gadget that implements a two-qubit interaction via momentum-dependent routing

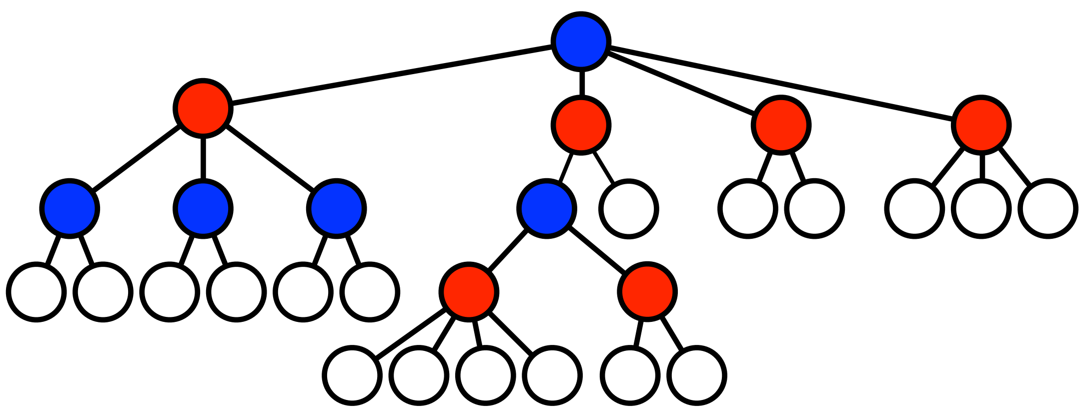


(+ extensive analysis to show the strategy works on a finite graph with small error)

Summary

Quantum walk is a powerful algorithmic tool.

Formula evaluation



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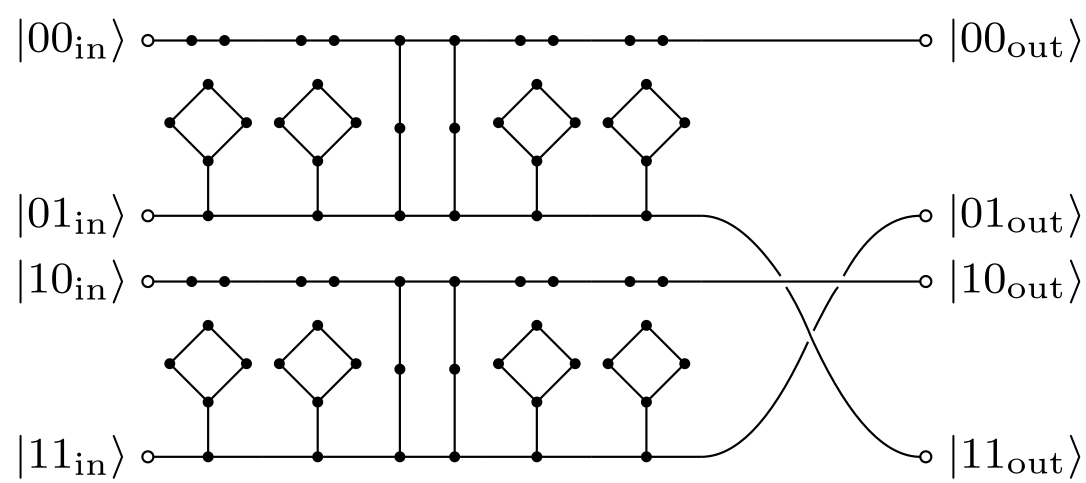

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- What other tools are useful for building quantum algorithms?
- What problems are hard even for quantum computers?