Problem Set #6

Quantum Error Correction
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Due Tues., Feb. 27, 2007

Problem #1. Repeating Syndrome Measurements

In this problem, we will consider different rules for repeating the error syndrome measurement for Shor error correction, and determine how many times the syndrome needs to be repeated in various methods.

a) First consider the case of a code which only corrects one error ($t = 1$). Suppose we repeat the syndrome measurement r times and take the result that appears most frequently. (If necessary, suggest a rule for how to decide when there is a tie for most frequent result.) What is the smallest r for which the error correction step satisfies EC properties 1 and 2?

b) Again for the $t = 1$ case, consider a procedure where we repeat until the measurement returns the same syndrome r times in a row. Now what value of r do we need to take to satisfy EC properties 1 and 2? Assuming there is only 1 error in the course of the full EC procedure, what is the maximum number of times we will need to measure the syndrome?

c) Answer the questions from parts a and b again, but now for a code with an arbitrary t.

Problem #2. Correcting a Bit Flip and a Phase Error

The 7 qubit code, as a CSS code, can correct both a bit flip and a phase error, even if those two errors are on different qubits. Therefore, in this problem, we will consider modifying the definition of the 1-filter so that it projects on the subspace of codewords with up to 1 bit flip error *and* up to 1 phase error, including when the bit flip and phase errors are on different qubits. (A state with an $X_1 Z_3$ error would pass this filter, but $Y_1 Z_3$ would not.) For simplicity, we assume a single error in the implementation of a fault-tolerant circuit still corresponds to an error in only one physical gate.

a) Show that Steane error correction for the 7-qubit code still satisfies EC properties 1 and 2 with this new definition of a 1-filter.

b) We showed in class that the full Clifford group could be performed transversally on the 7-qubit code. However, it turns out that not all transversal gates automatically satisfy gate properties 1 and 2 for the new definition of 1-filter. Give an example of a Clifford group gate that is not fault tolerant under the modified definition and explain why it is not fault tolerant.

c) Does the teleportation construction for fault-tolerant gates still work with the new definition? Can it be used to perform the Clifford group gates that failed to be fault tolerant in part b? Does your answer depend on the procedure used to prepare the ancillas; in particular, is there an ancilla preparation procedure that works for the old definition of the 1-filter, but not for the new definition?