When the Pope Was a Mathematician

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When the Pope was a Mathematician

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Probably nobody was surprised to read, of the newly elected Pope Benedict XVI, that he is a scholar of philosophy and theology. One expects a pope to have some familiarity with these subjects. Had the press proclaimed Benedict to be a mathematician they would have roused more interest. But only once in the history of the church (or of mathematics, if you prefer) has the Pope been a great mathematician, and that was a thousand years ago, before mathematics or the papacy entailed a lot of specialized knowledge. It's remarkable that the Pope on that remote occasion became Pope largely because of his fame as a mathematician. He was also an adventurous man, who deserves to be better known, whose life story can help to enliven courses on the history of mathematics, and highlight some of the cultural conditions that made the scientific revolution possible.

The man in question was Gerbert of Aurillac, who became Europe's leading mathematician (at the time a modest distinction) before reigning as Pope Sylvester II from 997–1003. Gerbert was born around 945 A.D. (the exact date is disputed [**1**, p. 456]) to absolutely obscure parents near Aurillac, a village due south of Paris and due west of Bordeaux, about 100 miles from the Mediterranean coast. This was the era some-

Figure 1. Gerbert was a French pope, therefore something of a national hero. There are no contemporary portraits, so this is as good a guess as any concerning his appearance. The French postal service didn't see fit to mention Gerbert's mathematical prowess.

times described as "the darkest hour of the Dark Ages [**2**, p. 309]," although, of course, darkness and light are very much in the eye of the beholder. The Islamic civilization from which Gerbert was to learn mathematics was then near its peak; to Muslims this would have been a very bright hour.

Times were dark enough in the vicinity of Aurillac, however; and Gerbert was fortunate to be able to enter the school attached to the nearby monastery of St. Gerald's, and to have as a teacher Brother Raymond, who exposed Gerbert to Latin classics (Terence, Cicero, Virgil) which provided him with the foundation of a humanist education. From time to time throughout his life, Gerbert expressed his devotion to Raymond in letters, (for example, "to him, after God, I thank above all mortals for whatever knowledge I possess" [**8**, let. 196]).[1] These classical Latin authors were the first of the influences that created the Gerbert phenomenon.

Raymond's instruction, however beneficial, had a glaring deficiency: it contained little or no mathematics. This was scarcely Raymond's fault; all of the mathematics known in Latin Europe at the time amounted to a handful of propositions asserted without proof, together with methods of addition and multiplication. These propositions were set forth in manuscripts (by Boethius and the Venerable Bede, among others) of which the most recent in Gerbert's day were already a couple of centuries old; their most important application was to calculate the date of Easter [**9**, p. 145]. Try to imagine a world today in which the most recent calculus text is Newton's *Methodus Fluxionum et Serierum Infinitarum,* and you will begin to have an idea of the intellectual decay and stagnation of the 10th century.

To comprehend this stagnation, it's worthwhile pausing to consider a manuscript from the period, a manuscript often copied, and hence presumably widely read: *Propositiones ad acuendos juvenes,* traditionally and uncertainly attributed to Alcuin of York (735–804). This text comprises 50 or so word problems, many of them repetitive (which befits a teaching text) and in no particular order (which doesn't). The most interesting (in the present author's opinion) is problem 42:[2]

> There is a ladder which has 100 steps. One dove sat on the first step, two doves on the second, three on the third, four on the fourth, five on the fifth, and so on up to the hundredth step. How many doves were there in all?

Alcuin's solution, anticipating Gauss, is to notice that there are 100 doves on the 1st and 99th steps, 100 more on the 2d and 98th, and so on for all the pairs of steps except the 50th and 100th. There is no attempt to generalize to arbitrary sums, and in fact the author has a peculiar obsession with the number 100, as noted below.

Problem 17, one of four river-crossing problems, reminds a modern reader of the tragic climax of a film noir:

> There were three men, each having an unmarried sister, all of whom needed to cross a river. Each man was desirous of his friend's sister. At the river they found only a small boat, in which only two persons at a time could cross. How did they cross the river, so that none of the sisters was defiled by the men?

The necessary condition that each man desires one, and only one, of the sisters is (in my inexpert opinion) clear in the Latin original, less clear in translation. In any

[1]This letter, written in the thick of a political crisis, contains the phrase "More tolerable is the clash of arms than the debates of laws"; indicating that in some ways Gerbert was himself a man of the Dark Ages.

[2]These translations are by Peter J. Burkholder; the entire document can be found at http://logica.ugent.be/albrecht/alcuin.pdf

case what's most remarkable about the problem is that the medieval author seems to find this indelicate situation completely ordinary.

Problem 43 has no solution and was "composed for rebuking" troublesome students:

> A certain man had 300 pigs. He ordered all of them slaughtered in three days, but with an uneven number killed each day. What number were to be killed each day?

One wonders how long the students needed to realize that three odd numbers can't add up to 300.

Problem 26 offers a glimmer of calculus:

> There is a field which is 150 feet long. At one end stood a dog, at the other, a hare. The dog chased the hare; the dog went nine feet per stride, the hare, seven. How many feet and how many leaps did the dog take before catching the hare?

The *Propositiones* reveal a society that retained only a few scraps of mathematical knowledge from the hellenistic period, and that valued those scraps chiefly as a way of preparing the mind for the more serious work of philosophy and theology.

Despite the small importance attached to it, Gerbert seems somehow to have become interested in what passed for mathematics at the time. Perhaps he spent days near a window of the monastery's shadowy library (candles would have been forbidden for fear of a conflagration), poring over manuscripts already ancient, impatient with the shortcomings of his texts. Or perhaps he learned everything in the course of an afternoon. What is fairly certain is that Gerbert learned all the mathematics available to him in Gaul, and wished for more. In this he was typical of medieval scholars: they were ignorant, but not stupid; stagnant, but not lazy; they were aware of their ignorance, and eager to remedy it when opportunity arose. In Gerbert's case the opportunity arose quite suddenly in the year 967, when St. Gerald's was visited unexpectedly by a traveling nobleman, Borel II, Count of Barcelona, and Duke of the Spanish March.

Borel (grandson of Wilfrid the Hairy, a name that could occur only in the darker parts of the Middle Ages) was traveling homeward, and his home, Barcelona, lay on the frontier of Moorish Spain, or Andalusia, the western extremity of the great Islamic civilization that stretched from the Pyrenees to Persia. The Arabs (as it is convenient to name this heterogeneous collection of peoples) inherited the scientific knowledge of ancient Greece, added to it, and spread the result throughout their empire. As poor as Latin Europe was in the knowledge of mathematics, so Andalusia was wealthy. And much of this wealth, perhaps all of it, would be available to a scholar living on the Andalusian border.

Thus Borel's visit presented Gerbert with a great opportunity, and Gerbert, with a little help from his abbot, seized it. The abbot inquired of Borel whether there were in Spain any very learned men, who might be willing to offer direction to a younger student. When Borel replied that indeed there were, the abbot promptly urged Borel to allow one of the monks of St. Gerald's to accompany him back to Spain, to study there under one of the Spanish masters. Borel agreed, and the brothers of St. Gerald's chose Gerbert, by acclamation, to be the lucky monk. Thus it was that Gerbert embarked on a journey of several hundred miles, of great discomfort and some danger, with no object at its end other than to study mathematics (a very fitting, although not entirely effective, example to set before students who complain of the injustice of having to walk across campus to attend a math class).

Figure 2. A 16th-century map of the Pyrenees and surrounding regions, with Aurillac in the upper right and the Catalonian cities of Barcelona and Vich, where Gerbert studied 967–970. (Printed with permission of Fr. Richard Blinn, S.J., www.ignatiushistory.info.)

Gerbert's master in Spain was Hatto, the bishop of Vich, a village near Barcelona, which was itself at that time a mere village of about a thousand people, sheltering behind old Roman walls. Questions surround the three years (967–970) Gerbert studied under Hatto. Did he cross the border into Andalusia, possibly visiting the great library at Cordova? How much did he learn from Latin sources (a lot, says the French scholar, eager to maintain that Gerbert's brilliance was a Gallic phenomenon), how much from Arabic sources (a lot, says everyone else), and how much did he himself create (everyone agrees not much). Whatever the source, by 970 Gerbert had learned enough to make him Europe's leading mathematician, a feat accomplished more easily then than now. Gerbert's Spanish studies were the second great intellectual influence on his life.

Given the relative poverty of students at any time, and the poverty of almost everybody in the 10th century, it's safe to suppose that after three years Gerbert had grown tired of student life, and was ready to look for a job. As luck would have it, Count Borel traveled to Rome in 970, hoping to establish his Spanish March as a diocese independent of France. It was natural that Gerbert, a learned churchman, should accompany him. Insofar as Christian Europe had a capital city, it was Rome, a perpetual meeting place for people with favors to grant or seek. Gerbert met and impressed leading Romans, including the Emperor, Otto I, and the Pope, John XIII, whom Otto had installed three years earlier amidst much bloodshed.

Otto offered Gerbert the post of tutor to his son, Otto II. This was as close to a tenured sinecure as the 10th century afforded, but the ever-adventurous Gerbert abandoned the position after a couple of years, and again the lure was a chance to study with a master. This time the master was a logician named Garamnus, living in Rhiems. Garamnus agreed to teach Gerbert logic in exchange for lessons in music and mathematics, topics Gerbert had mastered in Spain. But the Archbishop of Rhiems, Adalbero, interrupted this cozy arrangement almost before it began, and in the process launched Gerbert toward stardom, by entrusting him with the job of reforming and enlivening the cathedral school at Rhiems, which had fallen into decrepitude like almost everything else. (Cathedral schools were the remote ancestors of modern universities.)

Gerbert had a free hand to devise a curriculum to his liking; he did so, and ... "crowds of students came to him for instruction" and "the number of his disciples increased every day" because he "expounded the natural sciences ... by the use of the most wonderful instruments" [**11**, p. 18]. So great was the enthusiasm that even when, after ten years of teaching, Gerbert made a brief, disastrous attempt to run a monastery in Italy, the archbishop of Treves wanted to send students to him.

Gerbert succeeded because he combined elements of different cultures in surprising ways. He brought public speakers (the trial lawyers of their day) into his classroom, so that his students could sharpen their debating skills by practicing against professionals. Like his mentor Raymond, he taught things (Latin poets, Aristotle) that were elsewhere generally suppressed.

But the heart of Gerbert's success was his teaching of the mathematics that he had learned in Spain, of which the centerpiece was undoubtedly his discovery of Arabic numerals. These enabled Gerbert to streamline the abacus (see below) and to think expeditiously about number-problems in general. The advantages of Arabic numerals are obvious: Roman numerals suffice for counting, and lend an air of authority and antiquity to the most tawdry commercial enterprises, but they greatly inhibit doing arithmetic, or thinking about numbers. In a handful of surviving letters, Gerbert explained solutions to number-problems that perplexed his correspondents, who were possibly still grappling with Roman numerals. For example, given a list of "sesquiquartal superparticular numbers, ... resolve them into sesquitertian, then sesquialteral, then into three equal terms." Here *sesquiquartal* means that consecutive terms have the ratio $\frac{5}{4}$; *sesquitertial,* that the ratio is $\frac{4}{3}$; and *sesquialteral,* $\frac{3}{2}$. Superparticular numbers form a ratio $\frac{m+1}{m}$ for some positive integer m. Hence "sesquiquartal superparticular" is a redundancy. This problem appears in Boethius's *Arithmetic*; the terminology, and the method of solution, appear in the writings of Nichomacus of Gerasa, who in the 1st century wrote *An Introduction to Arithmetic*. Gerbert's solution begins with the particular case 16, 20, 25, and after a lengthy series of arithmetical steps he "resolves" these numbers into 9, 12, 16, then 4, 6, 9, and finally 1, 1, 1, as required.

It would be difficult to find a problem of less intrinsic interest to a student today; it was interesting to Gerbert, Boethius, and Nichomacus because the various ratios were associated with musical harmonies. But there are two other points of historical interest: first, although Gerbert solves only a particular case, it's obvious from his repeated insistence on an orderly execution of the required steps that he intends to provide a general solution (in fact, although Gerbert doesn't seem to realize it, any 3-element sesquiquartal list must be an integer multiple of Gerbert's initial list). Secondly, Gerbert's lengthy arithmetical procedures are much simpler and more intelligible in Arabic numerals.

Another specimen of Gerbert's mathematics is found in his famous letter to Adalbold, [**8**, let. 233] a mathematically-inclined cleric and teacher, who was puzzled to know whether the area of an equilateral triangle should be calculated geometrically as $\frac{bh}{2}$, or arithmetically as $\frac{n(n+1)}{2}$, the latter quantity being the number of dots in a equilateral triangular array. Gerbert plumps for the geometrical formula. This question is in essence a confusion about whether a number is a geometrical array of dots (the old Pythagorean idea, long since abandoned everywhere else in the world) or an abstraction capable of being assigned to different kinds of quantities. Gerbert's explanation is not very satisfactory [13, p. 224]; he assumes tacitly that $\sin(60°) = \frac{6}{7}$, and he seems to think that an equilateral triangle can be approximated with $\frac{n(n+1)}{2}$ squares, when in fact the resulting shape would be roughly isosceles, but not equilateral. However, he understands the relation between numbers and area ("for measuring areas we are accustomed to use only square feet," he writes), and it's reasonable to suppose that Arabic numerals may have helped to blow the Pythagorean nonsense out of his mind.

As noted above, Arabic numerals also enabled Gerbert to improve the old Roman abacus. Hitherto, the beads on an abacus were divided into columns, and the significance of a bead depended on its column, ones, tens, etc. Addition involved combining the appropriate numbers of beads; multiplication was a sequence of additions. Gerbert replaced the beads with counters made of horn, each counter having an Arabic numeral painted on its side. To add numbers, the abacist would replace their counters with a new counter bearing their sum.

Gerbert's earliest biographer claims that with Gerbert's abacus a student could "get the answer more quickly than he could express it in words" [3, p. 88]; one might say the Gerbertian OS entailed a big boost in processing speed. This was probably a star-struck exaggeration; for one thing, the abacist would probably from time to time spill the beads. But Gerbert's abacus was primarily a tool to teach arithmetic—he constructed a jumbo model for classroom demonstrations—and its real significance lay in the requirement that the user know the multiplication table, which in turn required or perhaps caused the recognition of multiplication and addition as arithmetical operations, rather than as practical rules for counting. Gerbert's abacus was a step towards mathematics as distinct from reckoning.

Gerbert exerted a similar clarifying influence on astronomy, which in the Middle Ages was bound up with religion. The perfectness of God was on display in the beauty and order of the heavens, and theories about the extent and composition of the universe implied things about the nature of God [9, pp. 303ff.]. Thus the potential for persecution of heterodox theorists was ever present, to be realized, tragically, in the cases of Galileo and others at the onset of the scientific revolution.

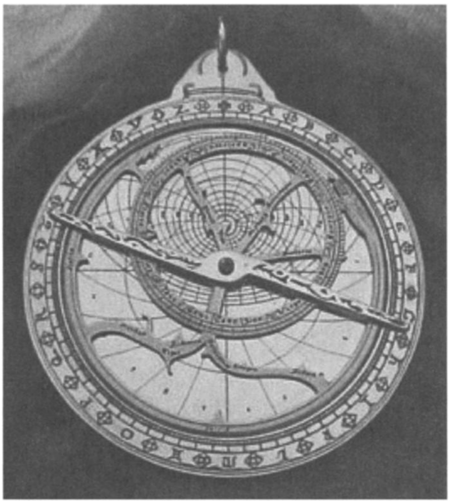
The evidence is incomplete, contained in just a few letters, but Gerbert seems to have taught astronomy as something close to scientific observation, an end in itself, apart from what it might reveal about the Creator. His contemporaries naturally found this puzzling, even sinister; and detractors claimed that he stayed up all night so that his trysts with the devil would go unnoticed.

What Gerbert was doing during these all-nighters was observing the night sky, assisted by one or another mechanical contrivance. These included the armillary sphere, the astrolabe, and a third, peculiar gadget of Gerbert's own invention.

The armillary sphere is a globe representing the earth surrounded by hoops representing the paths around the earth of selected planets or the Sun. In Gerbert's day it would have been made of wood; today it would be made of more durable materials. Gerbert used the armillary sphere to impart basic facts about astronomy [1, p. 468].

The astrolabe, a kind of round slide rule, could be used to fix precisely the times of heavenly events such as eclipses. In one form or another, it can be traced at least as far back as ancient Greece, but Gerbert introduced it to Christian Europe from Spain. In this way he forced medieval astronomers to associate numbers, in the form of time, with heavenly events—the beginning of mathematical astronomy in the Latin west [5, p. 72].

The astronomical gadget mentioned most often in Gerbert's letters is also the most obscure. Gerbert and his correspondents often mention "spheres," and sometimes describe them at length, but these descriptions are rather like recipes for omelets: if you



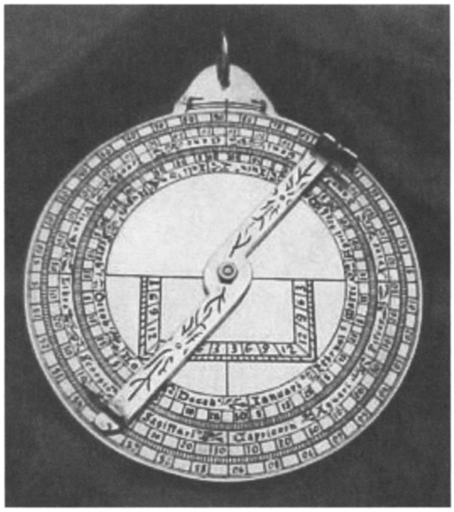


Figure 3. Front and back of a modern astrolabe. Gerbert's would have been less finely made, but essentially similar. (Printed with permission of Mr. Norman Greene, www.puzzlering.net.)

don't already know how to make one, the recipe won't help much. Gerbert's spheres seem to have been wooden, spherical (no surprise), marked with lines of altitude and azimuth, with tubes inserted diametrically so that, with one tube aimed at the northern pole star (a very faint object in the 10th century), the other tubes were trained on a sig-

Figure 4. A modern armillary sphere. (Printed with permission of R. Paselk, Humboldt State University.)

nificant celestial latitude such as the Arctic Circle. Gerbert seems to have constructed smaller spheres for classroom uses, and larger ones for observation. However, it's not known how large the spheres were, why he preferred spheres to hemispheres, how the tubes intersected at the center, or precisely what he observed.[3] Nonetheless, with a little imagination it's possible to see in Gerbert's spheres and tubes the remote ancestor of modern domed observatories [**8**, let. 2].

(Note the roundness of all these gadgets: Gerbert and his contemporaries took the roundness of the Earth for granted. The medieval belief in a flat Earth was an invention of the historians of the Renaissance, eager to belittle their immediate ancestors.)

To aid in the teaching of music, Gerbert constructed a water-powered organ. These marvelous devices were introduced to Europe from Byzantium in 757 A.D., and by the time of the Renaissance they had become fairly common. Instead of compressing air by means of a bellows, they used a flow of water through a long, vertical pipe to draw air under pressure into a chamber (the Aeolian chamber) which was connected to the windchest by a tube. The construction of such a device must have required great skills. Gerbert also made use of the monochord, a sounding-box with one string stretched tightly across. With it he demonstrated relationships between the length of a vibrating string and its musical pitch.

Gerbert made one lasting contribution to music: he named the notes of the scale (do, re, mi, fa, so, la—only six notes in his day) using the first syllables of the lines of a Latin hymn. The result was rather like "Doe, a deer," but with gravitas [**3**, p. 92].

As the years passed, and Gerbert's reputation for brilliance grew, he was gradually sucked into politics. Many scholars today ardently desire such a political detour, but the politics of the 10th century were a violent and treacherous sea, in which nearly every sailor eventually drowned. Gerbert bobbed up and down for ten years (many tedious details are hereby elided), much too busy for mathematics, until an alliance with the Emperor Otto II, son of Gerbert's old employer, led in 999 to Gerbert's election as Pope Sylvester II. Emperor and Pope dreamed of uniting East and West in a new Roman empire, like that of Constantine and Sylvester I in the 4th century. Nothing came of it; Otto died suddenly in 1002, and Gerbert followed the next year, possibly poisoned. Gerbert's political work bore little or no fruit.

What about his scholarly work? He was only minimally original; his revised abacus and the spheres with embedded observation tubes were his greatest innovations. His scholarly writing comprises a few expository essays and letters. But nobody in Christian Europe in his day was doing original work; there were no examples for him to follow, and no organized way of disseminating results. The Middle Ages were more interested in preserving knowledge than in adding to it. What was needed was not so much new information as a new way of handling information. In this regard Gerbert was most effective. He inspired his students with a love of learning and the exercise of reason (he wrote an essay on the latter topic). Many of his students later became bishops, with cathedral schools of their own where they attempted to replicate his methods. Other students of his took up influential posts in government, where they urged support for scholarship. One could easily overstate the case; Gerbert was not the first man of the Renaissance. But he was a humanist centuries before the word was coined; his life was the first sign of the spirit that gave life to the Renaissance and scientific revolution, and his inspiration was the love of mathematics.

[3] In a letter, he warns that the southern pole star can't be seen through any tube, because the Earth blocks the view—astronomy was an infant science in Gerbert's day.

vited to communicate with the publishers, who will correct the situation at the earliest opportunity.

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Wrong, Wrong, and Wrong: Math Guides Are Recalled

From *The New York Times*, March 25, 2005

Practice test questions given to fourth-grade teachers by the New York City Department of Education provided incorrect answers to several questions:

QUESTION The fifth-graders in Ms. Brown's class read 178 books in September, 212 books in October, and 254 books in November. If the students rounded the number they read each month to the nearest hundred to estimate the total, what would the total be?

A. 500 **B.** 600 **C.** 700 **D.** 800

Answer given in handout: **B**

QUESTION Evaluate the following expression if $x = 3$, $y = 5$, and $z = 0$:

$$xy - z + 2y$$

A. 14 **B.** 27 **C.** 59 **D.** 24

Answer given in handout: **D**

Besides the math mistakes, there were problems with grammar and spelling. For instance, the word "fourth" was misspelled on the cover of the fourth-grade manual.

