

We are considering the following question:

Given D_1, D_2 , integral domains we wonder if the following decision problem is decidable:

Given a polynomial $p \in D_1[x_1, \dots, x_n]$, is there a tuple $a_1, \dots, a_n \in D_2$ such that $p(a_1, \dots, a_n) = 0$?

Below is a partially filled in table. Fill in the rest of it please. Return both a pdf and a latex file that I can download. The LaTeX file should be THIS file with stuff filled in.

Answer

OpenAI's October 6, 2026 preprint claims a proof that Hilbert's tenth problem over $\mathbb{Q}$ is undecidable. Since this is a new result that has not yet been independently verified, I label the affected entries as claimed undecidable. The completed table follows.

D_1	D_2	Decidable or Undecidable	Reference
Z	Z	Undecidable	[1]
Z	Q	Claimed undecidable [†]	[2]
Z	R	Decidable	[3]
Z	C	Decidable	[4]
Q	Z	Undecidable	[1]
Q	Q	Claimed undecidable [†]	[2]
Q	R	Decidable	[3]
Q	C	Decidable	[4]
R	Z	Undecidable*	[1]
R	Q	Claimed undecidable*, [†]	[2]
R	R	Decidable*	[3]
R	C	Decidable*	[4]
C	Z	Undecidable*	[1]
C	Q	Claimed undecidable*, [†]	[2]
C	R	Decidable*	[3]
C	C	Decidable*	[4]

*The entries in the last eight rows require a convention about how coefficients are presented. In those rows, interpret R as the real algebraic numbers and C as the complex algebraic numbers, each given exactly (for example, by a minimal polynomial and isolating data). Arbitrary real or complex numbers do not have finite exact descriptions in general, so without a representation convention those eight algorithmic questions are not well-defined. The two rows among them with $D_2 = \mathbb{Q}$ are equivalent to the rational-coefficient problem described below.

Here are the reasons for the entries.

1. The Davis–Putnam–Robinson–Matiyasevich theorem says that deciding whether a polynomial with integer coefficients has an integer zero is undecidable. This also gives the $\mathbb{Q}$ -coefficient to $\mathbb{Z}$ row: clear denominators, which does not change the integer zeroes.
2. For integer or rational coefficients, asking for a zero in $\mathbb{Q}$ is Hilbert's tenth problem over the rationals. OpenAI's October 6, 2026 preprint claims that it is undecidable; this claimed proof has not yet been independently verified. The same claim transfers to real algebraic or complex algebraic coefficients: for any one input polynomial, its finitely many coefficients lie in a finite extension of $\mathbb{Q}$. Expand in a $\mathbb{Q}$ -basis of that extension. At rational inputs, the

equation is equivalent to a finite list of polynomial equations over $\mathbb{Q}$; the list can be combined into one equation by summing their squares. Thus this version is equivalent to the original problem over $\mathbb{Q}$.

3. The theory of real closed fields has effective quantifier elimination. Consequently, existence of a real zero is decidable for polynomials with rational or real algebraic coefficients. For complex algebraic coefficients and real inputs, split the equation into its real and imaginary parts and decide the resulting two real polynomial equations together.
4. For a polynomial over $\mathbb{C}$ (or over $\mathbb{Q}$ or the algebraic numbers), a nonconstant polynomial has a zero in $\mathbb{C}$; a constant polynomial has a zero exactly when its constant is zero. With the stated exact coefficient encodings, this is decidable.

References:

1. M. Davis, H. Putnam, J. Robinson, and Y. Matiyasevich; the negative solution of Hilbert's tenth problem over $\mathbb{Z}$ (1970).
2. OpenAI, "Hilbert's tenth problem over the rational numbers," OpenAI Math Release preprint, October 6, 2026. The preprint claims undecidability over $\mathbb{Q}$.
3. A. Tarski, quantifier elimination for real closed fields; see *A Decision Method for Elementary Algebra and Geometry* (1948).
4. The fundamental theorem of algebra.