HW 8 HONR 209M. Morally DUE Tuesday Mar 31

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? When is the final?

2. (40 points) (You may assume there is a 2-cut $(3, 4y, y)$-protocol and a 3-cut $(4, 6y, y)$-protocol.) Show there is a 4-cut $(5, 8y, y)$-protocol. Do ALL of the cases.

3. (60 points) In this problem we will guide you through a proof that there is NO 5-cut $(6, 9y, y)$-protocol.

   (a) Scenario: During a protocol for a 5-cut $(6, 9y, y)$ there is a piece P that EVERYONE thinks is $< 2y$. Alice will cut P into P_1, P_2 (we cannot control what she thinks of P_1, P_2 but we CAN assume that she thinks $P_1 \le P_2$). Show that we can set the opinions of Bob, Carol, Donna, Edgar, Frank such that NOBODY wants P_1 (NOTE that Alice won't want P_1 since $P < 2y$ and $P_1 \le P_2$ so $P_1 < y$). Note that hence we can conclude that if this scenario happens then the protocol cannot work since the protocol creates exactly 6 peieces, and one of them is bad for EVERYONE.

   (b) Scenario: During a protocol for $(6, 9y, y)$ there is a piece P that ALL BUT ALICE thinks is $< 2y$. Alice will cut P into P_1, P_2 (we cannot control what she thinks of P_1, P_2 but we CAN assume that she thinks $P_1 \le P_2$, though we won't be using that in this case). Show that we can set the opinions of Bob, Carol, Donna, Edgar, Frank such that none of them want P_1 or P_2. Note that hence we can conclude that if this scenario happens then the protocol cannot work since the protocol creates exactly 6 peieces, and two of them are bad for all but Alice. (Alice can take one of those pieces but whoever gets the other one will be unhappy.)

   (c) Scenario: During a protocol for $(6, 9y, y)$ there is a piece P that ALL BUT ALICE thinks is $< 2y$. Bob will cut P into P_1, P_2 (we cannot control what he thinks of P_1, P_2 but we CAN assume that he thinks $P_1 \le P_2$). Show that we can set the opinions of Alice, Carol, Donna, and Edgar, Frank such that NOBODY wants P_1 (NOTE that Bob won't want P_1 since $P < 2y$ and $P_1 \le P_2$ so

$P_1 < y$). Note that hence we can conclude that if this scenario happens then the protocol cannot work since the protocol creates exactly 6 peieces, and one of them is bad for EVERYONE.

(d) SUMMARY OF THE ABOVE THREE POINTS (This is not a question.) If you ever have a piece P such that all but at most one person thinks $P < 2y$, and P is the piece to be cut, then the protocol will fail. Hence in the future you can ignore this case. That will cut down on cases ALOT!

(e) PROVE there is no 5-cut $(6, 9y, y)$-protocol. HINT: On the first cut have Alice cut the cake into pieces P_1, P_2 such that Alice thinks $P_1 \geq P_2$ and Bob, Carol, Donna, Edgar, Frank all think $P_1 = 2y - \epsilon$ and $P_2 = 7y + \epsilon$. NOTE that by the above point, P_1 will never be cut again, which will help cut down the number of cases ALOT!