HON 209M, Midterm

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **You may use a Calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. **You must turn in your exam immediately when time is called at the end.**
3. There are 4 problems which add up to 100 points. The exam is 1 hour and 30 minutes.
4. In order to be eligible for as much partial credit as possible, show all of your work for each problem, **write legibly**, and **clearly indicate** your answers. Credit **cannot** be given for illegible answers.
5. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
6. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination*."

7. Fill in the following:

NAME :
SIGNATURE :
SID :
SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
TOTAL	

1. (30 points) For each of the following mark the statement as TRUE, FALSE, or UNKNOWN TO SCIENCE. 3 points for a correct response, -2 for an incorrect response. (ADVICE: Do not guess!)

   (a) For all $n \geq 2$ there is a discrete n-person protocol for proportional fair division that takes $\leq 100n \log_2 n$ cuts.

   (b) For all $n \geq 2$ there is a discrete n-person protocol for proportional fair division that takes $\leq 100n$ cuts.

   (c) For all $n \geq 2$ there is a discrete n-person protocol for envy-free fair division that takes $\leq 100n^3$ cuts.

   (d) For all $\epsilon > 0$, for all $n \geq 3$ there is a discrete n-person protocol such that, at the end, (1) everyone has within ϵ of $1/n$, and (2) one person has exactly $1/n$.

   (e) For all $n \geq 3$ there is a discrete n-person protocol such that, at the end, everyone has exactly $1/n$.

   (f) There exists a, b such that any discrete 2-person protocol for $(a : b)$ division requires at least 10^{100} steps.

   (g) There exists a, b such that any discrete 2-person protocol for $(a : b)$ division requires at least ab steps.

   (h) There is a discrete 3-person protocol for proportional cake cutting that uses at most 3 cuts.

   (i) There is a discrete 4-person protocol for proportional cake cutting that uses at most 4 cuts.

   (j) There is a discrete 5-person protocol for proportional cake cutting that uses at most 5 cuts.

2. (20 points) Consider the following protocol for dividing up a finite set of goods, some of which are fluid (can be split).

   (a) Alice and Bob agree on an ordering of the items which is NOT in terms of its value but in terms of which ones they are happiest splitting. For example, if the items were MONEY, GOLD, CAR then they MOST prefer to split MONEY, then second favorite to split is GOLD, but last favorite to split is CAR. So they would output (MONEY,GOLD,CAR).

   (b) Alice and Bob both allocate point values for all of the goods that add up to 100 (identical to the first stage of the Adjusted Winner Protocol). They CANNOT give two goods the same value.

   (c) The good are given out using ABBAABBAABBA.... That is, Alice gets the good that she values most, then Bob gets the good he values most that is available, then Bob gets the good that he values most that is available, then Alice gets the good that she values most that is available.

   (d) Alice and Bob both determine how many points worth of goods they got. We call the one who is better off *the better-off one*

   (e) If they do not have the same number of points then the good they are happiest splitting that the better-off one has is the one they split (similar to the Adjusted Winner Protocol).

   (The next two pages have two problems that use this protocol.)

Problem 1.1) Use this protocol in the following scenario: All items are splittable. Their preference for splitting is (Money,Gold,Ice Cream, Jewelery) meaning that they are happiest splitting Money and least happiest splitting jewelery.

Specify what each person gets and how many points they get.

Item	Alice	Bob
Money	40	40
Gold	30	20
Jewelery	20	10
Ice Cream	10	30

Problem 1.2) Assume Bob KNOWS Alice's point allocation. Say how he should dishonestly present his points so that he gets MORE than he got in part (1).

3. (30 points) For each of the following six situations either give a scenario where it happens or give a short explanation of why it can never happen. (there will be three on this page and two on the next page.

   (a) In the 3-person COME LATE protocol with Alice and Bob initially splitting the cake, Alice and Bob are envious OF EACH OTHER.

   (b) In the 3-person COME LATE protocol with Alice and Bob initially splitting the cake, Carol is envious of BOTH Alice and Bob.

   (c) In the 4-person TRIM protocol there is a person who envies EVERYONE else.

(d) In the 4-person Trim protocol there are two players who are envious of each other.

(e) In the 4-person Divide and Conquer protocol there are two players who are envious of each other.

4. (20 points) Show that, for all $\epsilon > 0$ there is a SIX-person protocol that will, given a cake, obtain an envy-free division of ALL BUT ϵ of it.

Scratch Paper