Homework 3, MORALLY Due Feb 17

1. (0 points but you have to answer) What is your name? Write it clearly. Staple your HW.

2. (10 points) In Led Zeppelin song *Stairway to Heaven* they say that profound line **All that Glitters is Not Gold**

   (a) What does this mean literally?

   (b) Write a quantified statement that captures what they intended.

3. (10 points) Abe Lincoln said **You can fool all of the people some of the time, and you can fool some of the people all of the time, but you can't fool all of the people all of the time.**

   Define domains and predicates so that you can express this statement and then express it in terms of quantifiers.

4. (20 points) Write the following in terms of quantifiers: **There is exactly one number that requires 9 cubes to sum to it– All of the rest can be done with 8.**

5. (20 points) Let D be a domain and $P(x)$ be a predicate over D. Write the following in terms of quantifiers: **There are exactly 5 elements of the domain for which P is true.**

6. (30 points) For each of the following sentences

   - Find a finite but nonempty domain where it is true OR prove that there is no such.
   - Find an infinite domain where it is true OR prove there is no such.

   (a) $(\forall x)(\forall y)(\exists z)[x < y \implies x < z < y]$

   (b) $(\forall x)(\exists y)[y^2 = x]$.

   (c) $(\forall x)(\exists y)[x \leq y]$.

   (d) $(\forall x)(\exists y)[x < y]$.

   (e) $(\exists y)(\forall x)[x < y]$.

   (f) $(\exists x)(\exists y)(\forall z)[x < y \wedge x \leq z \leq y]$.