Homework 7 Morally Due Mar 31
WARNING- THIS HW IS TWO PAGES LONG.

1. (0 points but you have to answer) What is your name? Write it clearly. Staple your HW. When is midterm 2?

2. (20 points) Prove by induction that for all $n \geq 1$

$$\sum_{i=1}^{n} i2^i = 2 + (n-1)2^{n+1}$$

3. (20 points) Let P be a statement about the integers. Assume we know the following:

   $P(4)$

   $(\forall n \in \mathsf{Z})[P(n) \implies P(n+3)]$.

   $(\forall n \in \mathsf{Z})[P(n) \implies P(n-3)]$.

   For which $n \in \mathsf{Z}$ do we know that $P(n)$ holds?

4. (20 points) You have an infinite supply of 4-cent coins and 5-cent coins. Note that you CAN create 9 cents (one 4-cent coin and one 5-cent coin) and 10 cents (two 5-cent coins) but you can't make 11 cents (proof: If you only use 4-cent coins then you can't make 11 since you can only make multiples of 4. If you only use 5-cent coins then you can't make 11 since you can only make multiples of 5. So you have to use at least one 4 and at least one 5. Hence you have 9. If you add 4 or 5 to that you exceed 11.)

   (a) Write a logical statement $P(n)$ for: "n coins can be created from 4-coins and 5-coins."

   (b) For $n = 4, 5, 6, \ldots$ see if n can be created from 4-coins and 5-coins. For each n that CAN be show how. For each n that CANNOT be prove why not. STOP when you are confident that you have an n_0 such that all $n \geq n_0$ CAN be.

   (c) For the value of n_0 that you determined in part (a), prove: *For all $n \geq n_0$ it is possible to create n cents with 4-coins and 5-coins.* (HINT: Use Strong Induction and you may need several base cases.)

(d) **Challenge Problem.** Suppose we were to have x-coins and y-coins. Would such an n_0 still exist? Determine a necessary condition that x and y must satisfy in order for this to happen.

5. (20 points) Give a Context Free Grammar for the language

$$\{a^n b^{2n} : n \in \mathsf{N}\}$$

6. (20 points) Show that the following Context Free Grammar generates $L_{a>b} = \{w : \#_a(w) > \#_b(w)\}$.

   ADJUSTMENT: LET G BE THE GRAMMAR AND $L(G)$ BE WHAT IT GENERATES. LET $L_{a>b} = \{w : \#_a(w) > \#_b(w)\}$. NEED ONLY PROVE THAT $L(G) \subseteq L_{a>b}$.

   $S \rightarrow aT$

   $S \rightarrow aS$

   $S \rightarrow bSS$

   $T \rightarrow aTb$

   $T \rightarrow bTa$

   $T \rightarrow TT$

   $T \rightarrow e$

   (HINT AND YOU CAN USE THIS: Note that T generates

   $Leq = \{w : \#_a(w) = \#_b(w)\}$

   (HINT: First prove that every derivation of a string of a's and b's of length n must have MORE a's than b's. This will be by induction. In doing this, pay attention to all possible FIRST rules in a derivation. Secondly, prove that every string of length n where there are more a's than b's can be generated. This is also by induction.)