Homework 10 Morally Due April 28

FOR THIS HW IF Y IS A SET OF NUMBERS THEN $SUM(Y)$ IS THE SUM OF THE ELEMENTS IN Y. For example, if $Y = \{2, 3, 7\}$ then $SUM(Y) = 12$.

1. (0 points) Where and when is the final?

2. (30 points) Let $X = \{1, \ldots, 20\}$.

   (a) Using the pigeonhole principle show that there are 22 subsets of X of size 3, which we denote $Y_1, \ldots, Y_{22}$, such that $SUM(Y_1) = \cdots = SUM(Y_{22})$.

   (b) Using the pigeonhole principle show that there are 24 subsets of X of size 3, which we denote $Y_1, \ldots, Y_{24}$, such that $SUM(Y_1) = \cdots = SUM(Y_{24})$. (HINT: You may want to remove some subsets and remove some sums.)

   (c) You did the last two problems with the Pigeon Hole Prin, hence you did not actually FIND triples with the same sum. Find and list out 30 triples that have the same sum. (For Fun but not to hand in: See how many triples you can find that have the same sum.)

3. (30 points)

   (a) Using the pigeonhole principle show that there are 90 subsets of X, which we denote $Y_1, \ldots, Y_{90}$, such that $SUM(Y_1) = \cdots = SUM(Y_{90})$.

   (b) Find numbers a, b such that ANY 3-coloring of the $a \times b$ grid has a monochromatic rectangle.

4. (40 points) Below we give sets of function from N to N¿ For each set say if it is FINITE or COUNTABLE or UNCOUNTABLE and PROVE it.

   (a) The set of functions f such that $(\forall x < y)[f(x) \le f(y)]$.

   (b) The set of functions f such that $(\forall x < y)[f(x) < f(y)]$.

   (c) The set of functions f such that $(\forall x < y)[f(x) \ge f(y)]$.

   (d) The set of functions f such that $(\forall x < y)[f(x) > f(y)]$.