Homework 11 Morally Due May 5

1. (0 points) Where and when is the final?

2. (40 points) For each of the following problems either find the a, b requested OR show that its NOT POSSIBLE. You must prove your answer in all cases.

   (a) Find an a, b with $a < b$ such that the following holds: For all 2-colorings of the $a \times b$ grid there exists at least TWO mono rectangles. (They do not have to be the same color but they could be.)

   (b) Find an a, b with $a < b$ such that the following holds: For all 2-colorings of the $a \times b$ grid there exists at least TWO mono rectangles OF THE SAME COLOR.

   (c) Find an a, b with $a < b$ such that the following holds: For all 2-colorings of the $a \times b$ grid there exists at least TWO mono rectangles OF DIFFERENT COLORS.

3. (30 points)

   (a) Show that there is no 1-1, onto, ORDER-PRESERVING function f from N to Q. (A function is ORDER-PRESERVING if $x < y$ implies $f(x) < f(y)$.

   (b) Show that there is no 1-1, onto, ORDER-PRESERVING (defined later) function f from Q to N. (A function is ORDER-PRESERVING if $x < y$ implies $f(x) < f(y)$.

4. (30 points) For each of the following relations R, answer and prove all of the following questions: (1) is R reflexive? (2) is R symmetric? (3) is R transitive?

   You must PROVE all of your assertions.

   (a) Relation is over the set R. $R(x, y)$ iff $|x - y| \geq 1$.
   (b) Relation is over the set R. $R(x, y)$ iff $x - y \in \mathsf{Q}$.
   (c) Relation is over the set Z. $R(x, y)$ iff x divides y.