Homework 2, MORALLY Due Feb 11

WARNING: THIS HW IS TWO PAGES LONG!!!!!!!!!!!!!!!!!

1. (25 points)

   (a) (15 points) Write a truth table with 3 inputs and 2 outputs for the following function:

   $f(x, y, z) = x + y + z \pmod 3$.

   (b) (10 points) Write a circuit for f using AND, OR, and NOT using the method shown in class (do not simplify- that would make it harder for the TA's to grade!)

2. (25 points)

   (a) (10 points) Use truth table so show that

   $$\neg(p \wedge q \wedge r) \equiv \neg p \vee \neg q \vee \neg r$$

   (This is called *DeMorgan's law on three variables.*)

   (b) (15 points) Consider the statement:

   $$\text{for all } n \ [\neg(p_1 \wedge \cdots \wedge p_n) \equiv \neg p_1 \vee \cdots \vee \neg p_n]$$

   Prove it. Note that you cannot use Truth Table since we want it for all n. Do not use Induction (later when we learn induction we will do that). Use reasoning about what the truth table for both sides must look like.

GO TO NEXT PAGE FOR MORE PROBLEMS!!!!!!!!!!!!!

3. (25 points – 5 points each) For each of the following statements write the negation without using any negations signs.

   (a) $x \leq 4$

   (b) $1 < x < 2$

   (c) $x_1 < x_2 < \cdots < x_n$

   (d) $x \leq 5$ OR $x \geq 10$

   (e) $x \leq 5$ AND $x \geq 10$

4. (25 points) Look up the lyrics to the song *If this isn't love* from the musical Finian's Rainbow. (You might also want to listen to it on You Tube.) Assume the song is true. For each simple proposition in the song assign a letter (try to make it mnemonic). For example

   L is *This is Love*

   so

   $\neg L$ is *This isn't Love*

   and

   W is *the whole world is crazy*

   Write down a proposition that expresses what happens if this isn't love. It will start:

   $$\neg L \Rightarrow (W \wedge \text{ other stuff}).$$

   (Note- one of the parts of the sentence doesn't make sense to me, we'll discuss later.)