

Homework 6, Morally due Tue Apr 2, 12:30PM
THE HW IS THREE PAGES LONG!!!!!!!!!!!!!!

1. (24 points) At the Twitty Family Reunion there are n people.
 - (a) Everyone hugs everyone. People even hug themselves! And Alice-hugs-Bob is counted as different from Bob-hugs-Alice. How many hugs are there?
 - (b) Everyone hugs everyone. Except that people do not hug themselves. Alice-hugs-Bob is counted as different from Bob-hugs-Alice. How many hugs are there?
 - (c) Everyone hugs everyone. Except that people do not hug themselves. Alice-hugs-Bob is counted as the same as Bob-hugs-Alice. How many hugs are there?
 - (d) Everyone hugs everyone. People even hug themselves! Alice-hugs-Bob is counted as the same as Bob-hugs-Alice. How many hugs are there?

2. (24 points)
 - (a) How many permutations are there of the letters in the sentence:
pack my box with five dozen liquor jugs
(ignore spaces, so the question is *packmyboxwithfivedozenliquorjugs*)
 - (b) How many permutations are there of the letters in the sentence:
Don't not ever stop not writing nothing
(ignore spaces as above)

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3. (28 points) Alice makes lunch for her darling. There is a sandwich- either PBJ, Turkey, Tomato, Egg salad, or Tuna fish, a fruit- either apple or blueberries or blackberries or a banana, and a snack- either pretzels, potato chips or applesauce.
- (a) How many ways can Alice make her darling lunch?
 - (b) If her darling does not like having apples and applesauce in the same lunch, then how many lunches can Alice make her?

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4. (21 points) (The first three parts are 0 points so they are really optional and there is nothing to hand in; however, you should do them for the enlightenment.) Let a_n be defined as follows

$$a_1 = 10$$

$$(\forall n \geq 2)[a_n = a_{\lfloor n^{3/4} \rfloor} + 20]$$

- (a) (0 points but you will need this for the next part) Write a computer program to compute, given n , a_1, \dots, a_n .
- (b) (0 points but you will need it for the next part) Compute a_i for $1 \leq i \leq 100,000$
- (c) (0 points) Based on your data make a good guess for the form of a good bound on a_n . (Do not look at the next question as it gives away the form.)
- (d) (21 points) Use constructive induction to find constants $A, B \in \mathbf{N}$ such that

$$(\forall n \geq 1)[a_n \leq A \lg n + B]$$