Homework 6, Morally due Tue Apr 2, 12:30PM

THE HW IS THREE PAGES LONG!!!!!!!!!!!!!

1. (24 points) At the Twitty Family Reunion there are n people.

   (a) Everyone hugs everyone. People even hug themselves! And Alice-hugs-Bob is counted as different from Bob-hugs-Alice. How many hugs are there?

   (b) Everyone hugs everyone. Except that people do not hug themselves. Alice-hugs-Bob is counted as different from Bob-hugs-Alice. How many hugs are there?

   (c) Everyone hugs everyone. Except that people do not hug themselves. Alice-hugs-Bob is counted as the same as Bob-hugs-Alice. How many hugs are there?

   (d) Everyone hugs everyone. People even hug themselves! Alice-hugs-Bob is counted as the same as Bob-hugs-Alice. How many hugs are there?

   SOLUTION TO QUESTION 1a: n^2.

   SOLUTION TO QUESTION 1b: $n^2 - n$.

   SOLUTION TO QUESTION 1c: $n(n-1)/2$.

   SOLUTION TO QUESTION 1d: $n(n-1)/2 + n$.

2. (24 points)

   (a) How many permutations are there of the letters in the sentence:
   pack my box with five dozen liquor jugs
   (ignore spaces, so the question is *packmyboxwithfivedozenliquorjugs*

   (b) How many permutations are there of the letters in the sentence:
   Don't not ever stop not writing nothing
   (ignore spaces as above)

   SOLUTION TO QUESTION 2

   a) The total number of letters in
   Pack my box with five dozen liquor jugs
   is 32.

Every letter appears once except:

3 o's

2 i's

2 e's

2 u's

So the answer is

$$\frac{32!}{3!2!2!2!}$$

b)

Omitted

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3. (28 points) Alice makes lunch for her darling. There is a sandwich- either PBJ, Turkey, Tomato, Egg salad, or Tuna fish, a fruit- either apple or blueberries or blackberries or a banana, and a snack- either pretzels, potato chips or applesauce.

   (a) How many ways can Alice make her darling lunch?

   (b) If her darling does not like having apples and applesauce in the same lunch, then how many lunches can Alice make her?

   SOLUTION TO QUESTION 3a

   $5 \times 4 \times 4 = 80$.

   SOLUTION TO QUESTION 3b

   We count how many lunches DO have both an apple and applesauce and then subtract from 80.

   $5 \times 1 \times 1 = 5$.

   So the answer is 75.

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4. (21 points) (The first three parts are 0 points so they are really optional and there is nothing to hand in; however, you should do them for the enlightnement.) Let a_n be defined as follows

   $a_1 = 10$

   $(\forall n \geq 2)[a_n = a_{\lfloor n^{3/4} \rfloor} + 20]$

   (a) (0 points but you will need this for the next part) Write a computer program to compute, given n, $a_1, \ldots, a_n$.

   (b) (0 points but you will need it for the next part) Compute a_i for $1 \leq i \leq 100,000$

   (c) (0 points) Based on your data make a good guess for the form of a good bound on a_n. (Do not look at the next question as it gives away the form.)

   (d) (21 points) Use constructive induction to find constants $A, B \in \mathsf{N}$ such that

$$(\forall n \geq 1)[a_n \leq A \lg n + B]$$

   Base Case: $a_1 = 5 \leq A \lg 1 + B = B$ so need $B \geq 5$

   IH: For all $n' < n$, $a_{n'} \leq A \lg(n') + B$

   IS:

$$a_n = a_{\lfloor n^{3/4} \rfloor} + 20 \leq a_{n^{3/4}} + 20 \leq A \lg \lg(n^{3/4})) + B + 20$$

   Need

$$A \lg(\lg(n^{3/4}) + B + 20 \leq A \lg \lg(n) + B$$

$$A \lg \lg(n^{3/4}) + 20 \leq A \lg \lg(n)$$

$$A(\lg((3/4) \lg(n)) + 20 \leq A \lg \lg(n)$$

$$A \lg(3/4) + A \lg(\lg(n)) + 20 \leq A \lg \lg(n)$$

$$A \lg(3/4) + 20 \leq$$

$$-A \lg(4/3) + 20 \leq$$

$$A \geq \frac{20}{\lg(4/3)} \sim 8.3$$

We take $A = 9$ and $B = 5$.