

Homework 10, Morally due Tue Apr 30, 3:30PM
THIS HW IS THREE PAGES!!!!!!!!!!

1. (0 points but if you don't show up to the final I will assume you got this problem wrong and you will get 0 points for this entire HW) WHEN IS THE FINAL? WHERE IS THE FINAL?

WHEN: Saturday May 18 4-6

WHERE: PHY 1201

2. (30 points)
 - (a) (15 points) Josh rearranges the letters in the sequence *machinery* randomly. What is the probability that the new sequence is *machinery*
 - (b) (15 points) Bill makes lunch for her darling. There is a sandwich- either PBJ, Turkey, Tomato, Egg salad, or Tuna fish, a fruit- either apple or blueberries or blackberries or a banana, and a snack- either pretzels, potato chips or applesauce. Suppose Bill selects a lunch to prepare uniformly at random out of all the possibilities. What is the probability that Bill's darling gets a lunch that DOES NOT have both an apple and applesauce.

SOLUTION TO PROBLEM ONE

1) There are $9!$ ways to arrange the letters, but only one of them is *machinery*. So the prob is $\frac{1}{9!}$

2) There are $5 \times 4 \times 3$ ways for Bill to make lunch for her darling. In how many ways DOES the lunch have both an apple and applesauce? Only 5 since the fruit and snack are already picked. Hence the prob that the lunch DOES have both an apple and applesauce is $\frac{5}{5 \times 4 \times 3} = \frac{1}{12}$. Hence the prob that the lunch does not have both is

$$1 - \frac{1}{12} = \frac{11}{12}.$$

Incidentally, Bill's darling actually does not mind having both an apple and applesauce.

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3. (40 points) I have two coins.

One of them is FAIR

One of them is BIASED: $\text{Prob}(H)=\frac{7}{12}$, $\text{Prob}(T)=\frac{5}{12}$.

One is chosen at random (prob 1/2 for each). That coin is tossed 20 times.

Do the following TWENTY ONE problems and put them in a table. For the first one show us your work (you can use a calculator or your program for the arithmetic), but the rest just have the answers in the table.

You will want to write a computer program for them. Note when the prob of biased goes from $> \frac{1}{2}$ to $< \frac{1}{2}$.

- The result is *HHHHHHHHHH* (so 20 H's and 0 T). What is the prob that the coin is biased?
- The result is *HHHHHHHHHT* (so 19 H's and 1 T). What is the prob that the coin is biased?
- The result is *HHHHHHHHTT* (so 18 H's and 2 T). What is the prob that the coin is biased?
- \vdots
- The result is *TTTTTTTTTT* (so 0 H's and 20 T). What is the prob that the coin is biased?

All numbers should be to six places, so for example

$$(7/12)^{20} \sim 0.000021$$

SOLUTION TO PROBLEM TWO

Let F be Fair and B be Biased.

$$P(B|H^{20}) = \frac{P(B) \times P(H^{20}|B)}{P(H^{20})}$$

$$P(B) = \frac{1}{2} = 0.5$$

$$P(H^{20}|B) = \left(\frac{7}{12}\right)^{20} \sim 0.000021$$

$$P(H^{20}) = P(H^{20}|B)P(B) + P(H^{20}|F)P(F) = \frac{1}{2}\left(\left(\frac{7}{12}\right)^{20} + \left(\frac{1}{2}\right)^{20}\right) = .000011.$$

Hence we get

$$0.5 \times (0.000021)(0.000011) = 0.9545454$$

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4. (30 points) I have two 10-sided die.

One of them is FAIR

One of them is BIASED: $\text{Prob}(1)=\text{Prob}(10)=\frac{1}{2}$ and $\text{Prob}(2)=\dots=\text{Prob}(9)=0$.

- (a) I roll the fair die. What is the expected value? What is the variance?
- (b) I roll the biased die. What is the expected value? What is the variance?
- (c) I roll both and add the values. What is the expected value? What is the variance?

SOLUTION TO PROBLEM THREE

- (a) Expected value = $\frac{11}{2}$. Variance = $\frac{33}{4}$.
- (b) Expected value = $\frac{11}{2}$. Variance = $\frac{81}{4}$.
- (c) Expected value = 11. Variance = $\frac{57}{2}$.