Homework 08, Morally due Mon Apr 19, 9:00AM

1. (0 points but if you miss the final that means you got this wrong retroactively and you will lose a lot of points). When is the FINAL? By what day do you need to tell Dr. Gasarch that you cannot make the midterm (if you cannot and know ahead of time)?

   HINT The TIME of the Final has been CHANGED. It is now Monday May 17, 8:00PM-10:15PM (We did this so that we could have that 15 minutes to deal with tech issues.)

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2. (30 points) In the country of Fredonia they use three kinds of coins: 11-cent and 35-cent and 72-cent.

   Let $P(n)$ be the statement

   n can be written as a sum of 11's, 35's and 72's.

   (In this problem you may want to write a program to help you; however, your answer should be readable without seeing or even knowing about the program. For example, you can't write: *the answer is 1987 since the program told me it was.*

   (a) (20 points) Find BY CONSTRUCTIVE INDUCTION a number n_0 such that

   $$(\forall n \ge n_0)[P(n) \rightarrow P(n+2)]$$

   (YES its $P(n) \rightarrow P(n+2)$.)

   (b) (10 point) Find n_1 such that

   - $(\forall n \ge n_1)[P(n)]$.
   - $\neg P(n_1 - 1)$

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3. (30 points) If you did not get full credit on problem 3 on the midterm and you get full credit on this problem, then you will get full credit on problem 3 on the midterm.

   In this problem you will prove the reciprocal theorem.

   (a) (0 points but you will need this later) Show that

   $$\frac{1}{2} + \frac{1}{3} + \frac{1}{8} + \frac{1}{42} + \frac{1}{56} = 1.$$

   (b) (0 points but you will need this later) Show that

   $$\frac{1}{56} = \frac{1}{58} + \frac{1}{1624}$$

   (c) (30 points) Use the first 2 parts of this problem to prove
   The Reciprocal Theorem
   For all $n \ge 3$ there exists $d_1 < \cdots < d_n \in \mathsf{N}$ such that $\sum_{i=1}^{n} \frac{1}{d_i} = 1$.
   (You may need to prove some early cases separately.)

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4. (40 points, 5 points each) For this problem the relations are forally subsets of $\mathsf{Z} \times \mathsf{Z}$.

   For this problem R means reflexive, S means symmetric, T means transitive.

   For the problems where we ask you to prove:

   When you claim a relation is R, prove it's R. Same for S and T.

   When you claim a relation is $\neg R$, prove it's $\neg R$ by giving a counterexample. Same for $\neg S$ and $\neg T$.

   (a) Give and prove a relation that is R,S,T that is NOT $=$.

   (b) Give a relation that is R,S,$\neg T$.

   (c) Give a relation that is R,$\neg S$,T that is NOT $\leq$ or $\geq$.

   (d) Give and prove a relation that is $\neg R$,S,T.

   (e) Give a relation that is R,$\neg S$,$\neg T$.

   (f) Give a relation that is $\neg R$,S,$\neg T$.

   (g) Give and prove a relation that is $\neg R$,$\neg S$,T that is NOT $<$ of $>$.

   (h) Give and prove a relation that is $\neg R$,$\neg S$,$\neg T$.