Homework 09, Morally due Mon April 26, 9:00AM

1. (0 points but if you miss the final that means you got this wrong retroactively and you will lose a lot of points). When is the FINAL?

   HINT Monday May 17, 8:00PM-10:15PM. This is NOT the time on the Universities formal schedule of finals.

2. (40 points) For this problem $n, k \in \mathsf{N}$.

   (a) (15 points) n people are in a line as in the hat game. An adversary puts hats on them that can be any of THREE colors, RED, WHITE, and BLUE. Ths usual rules: Person p_1 says a hat color, then p_2, ..., then p_n.

   There is an obvious strategy that does gets $n/2$ correct:

   If i is odd then p_i says p_{i+1}'s color, and p_{i+1} then knows his color.

   Give a strategy that gets BETTER THAN $n/2$ correct.

   IF $n = 1000$ then how many people get their hat color correct?

   (b) (10 points) n people are in a line as in the hat game. An adversary puts hats on them that can be any of k colors. Give a strategy that gets BETTER THAN $n/2$ correct.

   IF $n = 1000$ and $k = 10$ then how many people get their hat color correct?

   (c) (15 points) n people are in a line as in the hat game. An adversary puts hats on them that can be any of an infinite number of colors of hats (the people know the hat colors ahead of time). Give a strategy that gets BETTER THAN $n/2$ correct.

   IF $n = 1000$ then how many people get their hat color correct?

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3. (30 points, 10 points each) For each of the following sets say if it is FINITE, COUNTABLE or UNCOUNTABLE and prove your result. You may use that $\mathsf{N}, \mathsf{Z}, \mathsf{Q}$ are countable. You may use that (1) finite unions of countable sets are countable, (2) countable unions of countable sets are countable, (3) finite cross products of countable sets are countable.

   Domain is Dom for spacing purposes.

   (a) Set of all functions with dom N and co-dom PRIMES.

   (b) Set of all SURJECTION with dom N and co-dom $\{0, \ldots, 100\}$.

   (c) Set of all SURJECTION with dom $\{0, \ldots, 100\}$ and co-dom Q.

   (d) (0 points- THINK ABOUT) Set of all BIJECTIONS with dom PRIMES and co-dom N.

4. (30 points-15 points each) Let L_1 and L_2 be two linear orderings.

   An **Order-Preserving bijection between** L_1 **and** L_2 is a bijection f with domain L_1, co-domain L_2 such that

$$x < y \rightarrow f(x) < f(y).$$

   And now finally for our question.

   For each of the following pairs of linear orderings say if there is an order preserving bijection between them.

   If YES then describe the order-preserving bijection.

   If NO then prove there is no order-preserving bijection.

   (a) Q and $\mathsf{Q} + \mathsf{Q}$.

   (b) Z and $\mathsf{Z} + \mathsf{Z}$.