HW10 DUE Mon May 3, 9:00AM

For **Programming Problems** Send your code to Emily by email. Send the actual .java/.py/ect file. You need to use your .umd email address or it will not send. In your pdf, you must have the output your code provides. You can screenshot this or type it in. Hint: Use Python.

1. (0 points but if you miss the final that means you got this wrong retroactively and you will lose a lot of points). When is the FINAL? By what day do you need to tell Dr. Gasarch that you cannot make the midterm (if you cannot and know ahead of time)?

   HINT The TIME of the Final is Monday May 17, 8:00PM-10:15PM. This is NOT the time on the official School Schedule-of-finals.

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2. (40 point) In this problem we will guide you through the proof that e^2 is not rational. You may approximate $\frac{1}{(n+1)(n+2)\cdots(n+k)}$ by $\frac{1}{n^k}$ as we did in class.

   Note that if $e^2 = \frac{a}{b}$ then $be = ae^{-1}$. Hence, for all n,

$$bn!e = an!e^{-1}.$$

   (a) (10 points) Try to do the proof that e^2 is not rational in a manner similar to the proof that e is irrational. YOU WILL NOT SUCCEED. Tell us what went wrong.

   (b) (10 points) Show that, for large n there exists $D_1 \in \mathsf{N}$ and $0 < \delta_1 < \frac{1}{10}$ such that

$$bn!e = D_1 + \delta_1.$$

   (c) (10 points) Show that, for large even n there exists $D_2 \in \mathsf{N}$ and $0 < \delta_2 < \frac{1}{10}$ such that

$$an!e^{-1} = D_2 - \delta_2.$$

   (d) (10 points) Prove that e^2 is irrational. You should use the preveious 2 parts in your proof.

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3. (30 points) In the proof that e is transendental we used the approximation

$$\sum_{i=1}^{\infty} \frac{1}{(n+1)\cdots(n+i)} = \frac{1}{n-1}$$

In this problem we will see how good this approximation is.

(a) Write a program that will, on input n, L
   - Compute $\sum_{i=1}^{L} \frac{1}{(n+1)\cdots(n+i)}$.
   - Compute $\frac{1}{n-1}$.
   - Compute the difference of these two quantifites which we denote ϵ.

(b) Run your program on all $50 \le n \le 100$ with $L = 1000$. Graph n vs ϵ.

   Does it look like a good approximation?

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4. (30 points) Recall the set ONEFOUR from a prior HW:

$$\{n : n \equiv 1 \pmod 4\}.$$

Recall also that some numbers are prime-in-ONEFOUR but not prime normally, such as 9.

FIND a number in ONEFOUR that factors into primes-in-ONEFOUR in MORE THAN one way. (Hence Unique Factorization does not hold in ONEFOUR.)

You may use a program for this, though there is a way to find such a number without a program