Bayes Theorem

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- $\Pr[A|B] = \Pr[B|A] \cdot \frac{\Pr[A]}{\Pr[B]}$

Note: This is very useful in both this course and in life.

Example of Application of Bayes's theorem

$\Pr[A|B] = \Pr[B|A] \cdot \frac{\Pr[A]}{\Pr[B]}$. There are two coins:

1) Coin F is fair: $\Pr(H) = \Pr(T) = \frac{1}{2}$.
2) Coin B is biased: $\Pr(H) = \frac{3}{4}$, $\Pr(T) = \frac{1}{4}$.

Alice picks coin at random, flips 10 times, gets all H.
Is the coin definitely biased?

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What is Prob that it is biased? VOTE:

1. Between 0.99 and 1.0
2. Between 0.98 and 0.99
3. Between 0.97 and 0.98
4. Less than 0.97

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We will see that it is 0.982954, so between 0.98 and 0.99.

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$$\Pr(B|H^{n}) = \frac{1}{1 + (2/3)^{n}}.$$