Combinatorics

250H

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Proof (1): The number of subsets of $\{1, 2, \ldots, n\}$ is 2^n. From that set we can choose 0 elements or 1 elements or … or n elements.

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Proof (2): Consider the identity, $(x + y)^n = \Sigma\, C(n, i)\, x^i\, y^{n-i}$

Choose $x = y = 1$. Now we have $(1 + 1)^n = \Sigma\, C(n, i)\, 1^i\, 1^{n-i}$ or $2^n = \Sigma\, C(n, i)$.

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Option 1: Lets ignore gender. Then we have a total of $2n$ people.

So we have $C(2n, n)$ ways.

There are *n* boys and *n* girls. We want to pick out *n* people. How many ways can we do this?

Option 1: Lets ignore gender. Then we have a total of 2*n* people.

So we have C(2*n*, *n*) ways.

Option 2: We can pick 0 girls and *n* boys, 1 girl and *n*-1 boys, ..., *n* girls and *n*-1 boys.

So we have $\Sigma\, [C(n, i)]^2$ ways.

There are n boys and n girls. We want to pick out n people. How many ways can we do this?

Option 1: Lets ignore gender. Then we have a total of $2n$ people.

So we have $C(2n, n)$ ways.

Option 2: We can pick 0 girls and n boys, 1 girl and n-1 boys, ..., n girls and 0 boys.

So we have $\Sigma\, [C(n, i)]^2$ ways.

This is another identity: $\Sigma\, [C(n, i)]^2 = C(2n, n)$

Combinatorial Identities

1. $(x+y)^n = \Sigma\ C(n, i)\ x^i\ y^{n-i}$

2. $\Sigma\ [C(n, i)]^2 = C(2n, n)$