

# Homework 10

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250H

(30 points) Recall the set ONEFOUR from a prior HW:

$$\{n : n \equiv 1 \pmod{4}\}.$$

Recall also that some numbers are prime-in-ONEFOUR but not prime normally, such as 9.

FIND a number in ONEFOUR that factors into primes-in-ONEFOUR in MORE THAN one way. (Hence Unique Factorization does not hold in ONEFOUR.)

You may use a program for this, though there is a way to find such a number without a program

If  $x \equiv 3 \pmod{4}$  and  $y \equiv 3 \pmod{4}$  then  
 $xy \equiv 3 \times 3 \equiv 1 \pmod{4}$ .

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Hence numbers of the form  $xy$  are primes in ONEFOUR.

We multiply together four such numbers:

$$3 \times 7 \times 11 = 3465$$

Note that

- $3465 = 4 \times 4 \times 866 + 1$ , hence 3465 is in ONEFOUR.
- $3465 = (3 \times 7) \times (11) = 21 \times 165$ . Both 21 and 165 are PRIMES IN ONEFOUR.
- $3465 = (3 \times 11) \times (7) = 33 \times 105$ . Both 33 and 105 are PRIMES IN ONEFOUR.

Hence 3465 factors two different ways.