Making Change

250H

Problem:

How many ways can you make change of $1.00 with pennies, nickels, dimes and quarters?

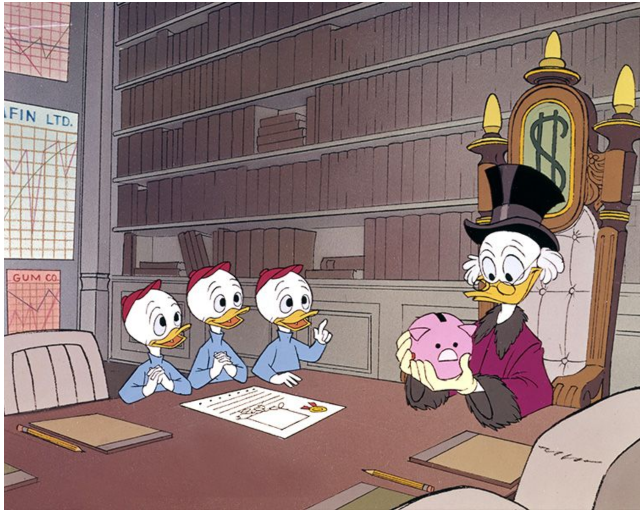
Easier Question: How many ways can you make change of $0.16 with pennies, nickels, dimes, quarters?

How many ways can you make change of $0.16 with pennies, nickels, dimes, quarters? 6 ways

1d	+	1n	+	1p	1(10) + 1(5)+ 1(1)	16
1d	+			6p	1(10) + 6(1)	16
		3n	+	1p	3(5) + 1(1)	16
		2n	+	6p	2(5) + 6(1)	16
		1n	+	11p	1(5) + 11(1)	16
			+	16p	16(1)	16

How many ways can you make change of $1.00 with pennies, nickels, dimes and quarters?

- Discuss in Breakout Rooms
  - 10 mins
- Stop being antisocial and talk to your classmates
  - You can blame Bill for this one
  - I'd never be so evil as to make you talk to people



How many ways can you make change of $1.00 with pennies, nickels, dimes and quarters?

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2. b_n is the number of ways to make change of n cents using the first two coins (Pennies and s-sent coins). $(\forall n)[b_n = a_n + b_{n-s}]$. We use that $(\forall n \leq -1)[a_n = 0]$.

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3. c_n is the number of ways to make change of n cents using the first three coins (Pennies, s-cent coins, and t-cent coins). $(\forall \text{n})[c_n = b_n + c_{n-t}]$.

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4. d_n is the number of ways to make change of n cents using all four coins (pennies, s-cent coins, t-cent coins, and u-cent coins). $(\forall n)[d_n = c_n + d_{n-u}]$.

How many ways can you make change of $1.00 with pennies, nickels, dimes and quarters?

Another way to ask this question: Compute d_{100}.

$d_n = c_n + d_{n-25}$

$d_{100} = c_{100} + d_{75}$

$d_{100} = c_{100} + c_{75} + d_{50}$

$d_{100} = c_{100} + c_{75} + c_{50} + d_{25}$

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Another way to ask this question: Compute d_{100}.

$c_0 = 1$

$c_{25} = b_{25} + b_{15} + b_5$

$b_{25} = a_{25} + a_{20} + b_{15} = 6$

$b_{15} = a_{15} + a_{10} + b_5 = 4$

$b_5 = a_5 + b_0 = 2$

$c_{25} = b_{25} + b_{15} + b_5 = 6 + 4 + 2$

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$d_{100} = c_{100} + c_{75} + c_{50} + c_{25} + c_0$

$c_0 = 1$

$c_{25} = b_{25} + b_{15} + b_5 = 6 + 4 + 2 = 12$

$c_{50} = b_{50} + b_{40} + b_{30} + b_{20} + b_{10} + b_0 = 11 + 9 + 7 + 5 + 3 + 1 = 36$

$c_{75} = b_{75} + b_{65} + b_{55} + b_{45} + b_{35} + c_{25} = 16 + 14 + 12 + 10 + 8 + 12 = 72$

$c_{100} = b_{100} + b_{90} + b_{80} + b_{70} + b_{60} + c_{50} = 21 + 19 + 17 + 15 + 13 + 36 = 121$

How many ways can you make change of $1.00 with pennies, nickels, dimes and quarters? 242 ways

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$$d_{100} = 1 + 12 + 36 + 72 + 121 = 242$$

How can we code this?

How many ways can you make change of $0.16 with pennies, nickels, dimes, quarters? 6 ways

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- Brute force
- Bad Recursion
- Good Recursion

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How can we code this efficiently?

- Using Dynamic Programing!
- Dynamic Programming is used to optimize something that uses recursion
- We store the results of subproblems so we do not have to recompute the same thing later
- Ex: If we already computed b_5, we would waste time recomputing it
  - $b_5 = a_5 + b_0 = 2$
  - $b_{15} = a_{15} + a_{10} + [a_5 + b_0] = a_{15} + a_{10} + b_5 = a_{15} + a_{10} + 2$

```
makeChange(n):
    S = [1, 5, 10, 25]
    matrix = [n+1][4]
    for i =0 to 4
        table[0][i] = 1
    for i = 1 to n+1
        for j=0 to 4
            if i-S[j] >= 0 { x = matrix[i - S[j]][j] }
            else { x=0 }
            if j >= 1 {y = table[i][j-1]}
                else { y=0 }
            table[i][j] = x + y
    return table[n][m-1]
```