Review for Midterm

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2. $(\forall x)(\exists y)[xy = 1]$.

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Truth Table (TT) Problem

Consider the following arithmetic function:

$$f(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \begin{cases} T & \text{if exactly ONE input is T} \\ F & \text{otherwise} \end{cases} \tag{1}$$

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LOTS of rows are T, actually $\binom{n}{n/2} \sim \frac{2^n}{\sqrt{n}}$.

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$$4 = x^3 + y^3 + z^3.$$

$x, y, z \leq 1$ since if (say) $x \geq 2$ then $4 \geq 2^3 = 8$ which is not true.
Hence $x^3 + y^3 + z^3 \leq 3 < 4$.

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$$\binom{x}{x'}\binom{y}{y'}$$

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Neither is on the subcommittee
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So the answer is

$$\binom{x-1}{x'}\binom{y-1}{y'-1}+\binom{x-1}{x'-1}\binom{y-1}{y'}+\binom{x-1}{x'}\binom{y-1}{y'}.$$

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A **Straight Flush** is 4 cards that are both a straight and a flush. For example $(3, S_1)$, $(4, S_1)$, $(5, S_1)$, $(6, S_1)$.

More Kenny Rogers

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More Kenny Rogers

d) Give (r, s) with $r, s \geq 5$ so that if r ranks and s suits then the prob(flush)<prob(straight).

$$s\binom{r}{4} - rs < rs^4 - rs$$

$$\frac{r!}{4!(r-4)!} < rs^3$$

$$\frac{r(r-1)(r-2)(r-3)}{4!} < rs^3$$

$$\frac{(r-1)(r-2)(r-3)}{4!} < s^3$$

Suffices to make $r^3 < s^3$, so $r < s$.

More Kenny Rogers

e) Give (r, s) with $r, s \geq 5$ so that if r ranks and s suits then the prob(flush)>prob(straight).

$$s\binom{r}{4} > rs^4$$

$$\frac{r!}{4!(r-4)!} > rs^3$$

$$\frac{(r-1)(r-2)(r-3)}{4!} > s^3 \text{ Suffices to make}$$

$$\frac{(r-3)^3}{24} > s^3$$

$$(r-3)^3 > 24s^3 = (24^{1/3}s)^3$$

$$r - 3 > 24^{1/3}s \text{ Suffice to take } r - 3 > 3s$$

Take $r = 19$ and $s = 5$.