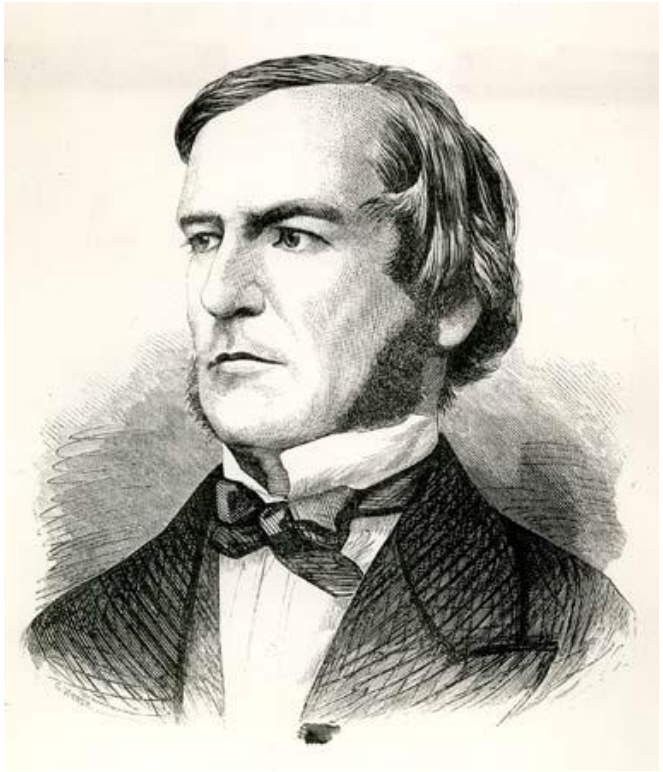
START

RECORDING

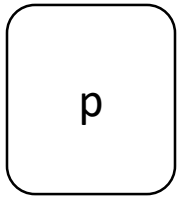
# Module 1: Propositional Logic

- The most elementary kind of logic in Computer Science
- Also known as Boolean Logic, by virtue of *George Boole* (1815 – 1864)



# Propositional Symbols

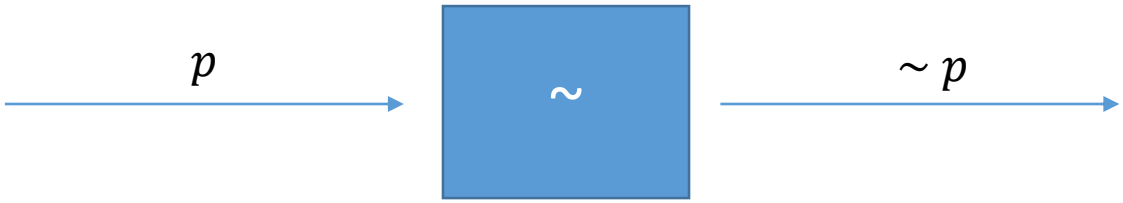
- The building blocks of propositional logic.
- Think of them as **bits** or **boxes** that hold a value of 1 (True) or 0 (False)
- Denoted using a lowercase english letter (p, q, … , a)



# Operations in boolean logic

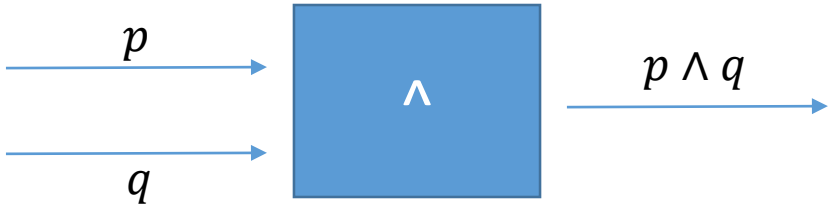
- There are three basic operations in boolean logic
  - Conjunction (AND)
  - Disjunction (OR)
  - Negation (NOT)
- Other operations can be defined *in terms of those three.*

# Negation $(\mathbf{NOT}, \sim, \neg)$



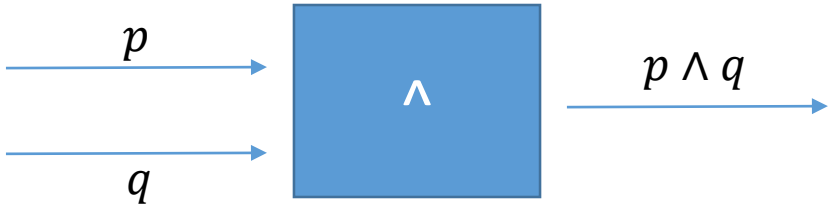

| $p$ | $\sim p$ |
|---|---|
| ***F*** | ***T*** |
| ***T*** | ***F*** |

# Conjunction (^)




| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

# Conjunction (^)




| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

Rule of thumb: p **and** q must be 1

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | ***?*** |
| ***F*** | ***T*** | ***?*** |
| ***T*** | ***F*** | ***?*** |
| ***T*** | ***T*** | ***?*** |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | |
| ***F*** | ***T*** | |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \wedge (\sim q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***F*** |

# Disjunction


| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

# Disjunction


| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

Rule of thumb:
**one of** p **or** q
must be 1

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***?*** |
| ***F*** | ***T*** | ***?*** |
| ***T*** | ***F*** | ***?*** |
| ***T*** | ***T*** | ***?*** |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | |
| ***F*** | ***T*** | |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

- Anything interesting here?

# Fun exercise

- Fill-in the following truth table:

| $p$ | $q$ | $p \vee (p \wedge q)$ |
|---|---|---|
| F | F | F |
| F | T | F |
| T | F | T |
| T | T | T |

- Anything interesting here?

# Implication ($\Longrightarrow$)

- "If –then"

| $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|
| ***F*** | ***F*** | ***T*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

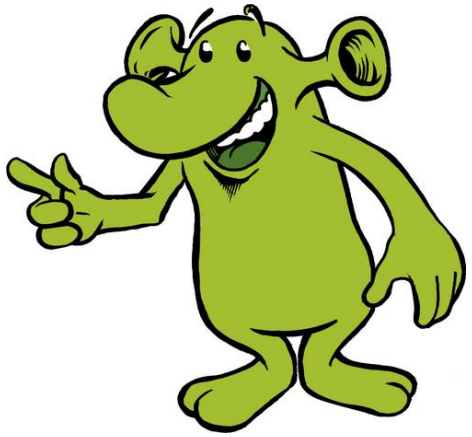
# Implication ($\Longrightarrow$)

- "If –then"

| $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|
| ***F*** | ***F*** | ***T*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

# Gorslax learns about birds

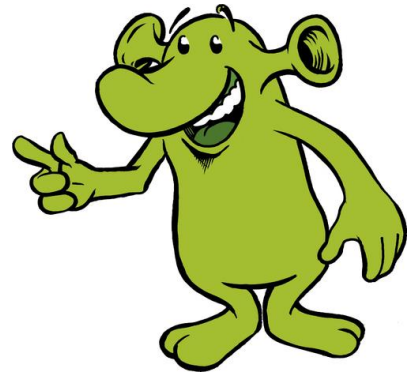
- Gorslax, an alien from the Andromeda Galaxy, visits planet Earth on a scientific expedition.
- Gorslax's planet has a very strong gravitational field which does not allow for the evolution of aviary life.
  - So he starts studying Earth's birds.



# Gorslax learns about birds

# Gorslax learns about birds

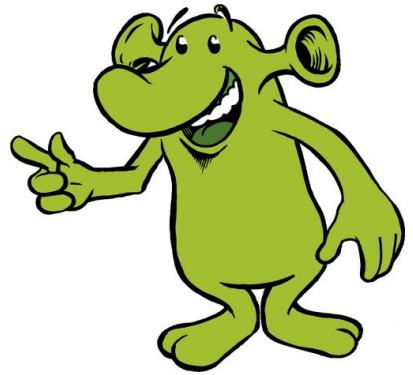
Hmmm… **From what I see until now**, it seems as if **all birds fly**



| $bird$ | $flies$ | $bird \Rightarrow flies$ |
|---|---|---|
| *F* | *F* | *T* |
| *F* | *T* | *T* |
| *T* | *F* | *F* |
| *T* | *T* | *T* |

# Gorslax learns about birds

Well **this thing clearly doesn't fly**, but it's also **not a bird**, so **I don't care**; I **still believe that all birds fly!**



| $bird$ | $flies$ | $bird \Rightarrow flies$ |
|---|---|---|
| *F* | *F* | *T* |
| *F* | *T* | *T* |
| *T* | *F* | *F* |
| *T* | *T* | *T* |

# Gorslax learns about birds


| $bird$ | $flies$ | $bird \Rightarrow flies$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

# Gorslax learns about birds

Whoops! **Here's at least one bird that doesn't fly**! So my syllogism ***"if bird then flies"*** does not **universally** apply!

| $bird$ | $flies$ | $bird \Rightarrow flies$ |
|---|---|---|
| *F* | *F* | *T* |
| *F* | *T* | *T* |
| *T* | *F* | *F* |
| *T* | *T* | *T* |

# Bi-conditional ($\Leftrightarrow$)

- "If and only if"

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| ***F*** | ***F*** | ***T*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

# Practice

- Fill in the following truth tables:

| $p$ | $p \Longrightarrow (\sim p)$ |
|---|---|
| ***F*** | ***?*** |
| ***T*** | ***?*** |

| $p$ | $q$ | r | $(p \wedge q) \Rightarrow r$ |
|---|---|---|---|
| ***F*** | ***F*** | ***F*** | ***?*** |
| ***F*** | ***F*** | ***T*** | ***?*** |
| ***F*** | ***T*** | ***F*** | ***?*** |
| ***F*** | ***T*** | ***T*** | ***?*** |
| ***T*** | ***F*** | ***F*** | ***?*** |
| ***T*** | ***F*** | ***T*** | ***?*** |
| ***T*** | ***T*** | ***F*** | ***?*** |
| ***T*** | ***T*** | ***T*** | ***?*** |

# Contradictions / Tautologies

- Examine the statements:
  - $p \wedge (\sim p)$
  - $p \vee (\sim p)$
- What can you say about those statements?

# What if T=1 and False =0?

- This is useful when we get to circuits
- What is AND, OR, and NOT?
- NOT = 1-x

| $x$ | $\sim x$ |
|---|---|
| ***F*** | ***T*** |
| ***T*** | ***F*** |

| $x$ | $1-x$ |
|---|---|
| ***0*** | ***1*** |
| ***1*** | ***0*** |

# What if T=1 and False =0?

- What is AND, OR, NOT?
- AND = xy

| $x$ | $y$ | $x \wedge y$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***F*** |
| ***T*** | ***F*** | ***F*** |
| ***T*** | ***T*** | ***T*** |

| $x$ | $y$ | $xy$ |
|---|---|---|
| ***0*** | ***0*** | ***0*** |
| ***0*** | ***1*** | ***0*** |
| ***1*** | ***0*** | ***0*** |
| ***1*** | ***1*** | ***1*** |

# What if T=1 and False =0?

- What is AND, OR, and NOT?
- OR = x+y? NO!

| $x$ | $y$ | $x \vee y$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

| $x$ | $y$ | $x + y$ |
|---|---|---|
| ***0*** | ***0*** | ***0*** |
| ***0*** | ***1*** | ***1*** |
| ***1*** | ***0*** | ***1*** |
| ***1*** | ***1*** | ***10*** |

# What if T=1 and False =0?

- What is AND, OR, and NOT?
- OR = x+y-xy

| $x$ | $y$ | $x \vee y$ |
|---|---|---|
| ***F*** | ***F*** | ***F*** |
| ***F*** | ***T*** | ***T*** |
| ***T*** | ***F*** | ***T*** |
| ***T*** | ***T*** | ***T*** |

| $x$ | $y$ | $x + y$ | $x + y - xy$ |
|---|---|---|---|
| ***0*** | ***0*** | ***0*** | ***0*** |
| ***0*** | ***1*** | ***1*** | ***1*** |
| ***1*** | ***0*** | ***1*** | ***1*** |
| ***1*** | ***1*** | ***10*** | ***1*** |

STOP

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