Homework 1, MORALLY Due Feb 1 at 9:00AM, DEAD-CAT DAY Feb 3, 9:00AM

1. (10 points) When is the untimed part of the first midterm MORALLY DUE (give date and time)?

   When is the timed part of the first midterm (give date and time)?

   When is the untimed part of the second midterm MORALLY DUE (give date and time)?

   When is the timed part of the second midterm (give date and time)?

   If you cannot make timeslot of the first midterm TELL DR. GASARCH NOW!!!!!!

   If you cannot make timeslot of the second midterm TELL DR. GASARCH NOW!!!!!!

SOLUTION

UNTIMED MIDTERM ONE: Morally Due Feb 28 at 9:00AM.

TIMED MIDTERM ONE: March 9 at 8:00PM.

UNTIMED MIDTERM TWO: Morally Due April 12 9:00AM.

TIMED MIDTERM TWO: April 20, 8:00PM.

END OF SOLUTION

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2. (20 points) Give a Propositional Formula on three variables that has exactly four satisfying assignments. Give the satisfying assignments.

SOLUTION

$(x_1 \wedge x_2 \wedge x_3) \vee (\neg x_1 \wedge x_2 \wedge x_3) \vee (\neg x_1 \wedge \neg x_2 \wedge x_3)(\neg x_1 \wedge \neg x_2 \wedge \neg x_3)$

The satisfying assignments are (T, T, T) and (F, T, T) and (F, F, T), (F, F, F).

END OF SOLUTION

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3. (20 points)

- Do a truth table for $(p \implies q) \implies r$.
- Do a truth table for $p \implies (q \implies r)$.
- Are they equivalent? If NOT then state a row where they differ.

SOLUTION

SHORT CUT: Recall that the only way that $x \implies y$ is F is if x is T and y is F.

The only way $p \implies (q \implies r)$ is F is if p is T and $(q \implies r)$ is F. The latter can only happen if q is T and r is F. Hence the only way $p \implies (q \implies r)$ is F is if p is T, q is T, and r is F.

The only way $(p \implies q) \implies r$ is F is if $(p \implies q)$ is T and r is F. $(p \implies q)$ is T when (p, q) is either (T, T), (F, T) or (F, F). Hence $(p \implies q) \implies r$ is F when (p, q, r) is either (T, T, F), (F, T, F), (F, F, F).

p	q	r	$p \implies (q \implies r)$	$(p \implies q) \implies r$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

NOT equiv: they differ on the row (F,T,F).

END OF SOLUTION

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4. (25 points) n has the EMILY property if there is a formula on n variables with exactly $n^3 + 17$ satisfying assignments.

   (a) (10 points) Fill in the following sentence n has the EMILY property IFF BLANK(n). The condition BLANK has to be simple, for example, n is divisible by 5 (thats not the answer).

   (b) (15 points) Prove the statement you made in the first part. Note that this means you have to show that

   If BLANK(n) then n has the EMILY property

   and

   If NOT(BLANK(n)) then n DOES NOT have the EMILY property

SOLUTION

a) A formula on n variables has at most 2^n satisfying assignments. Hence you need $n^3 + 17 \le 2^n$. We need that to be simple.

n	$n^3 + 17$	2^n
1	18	2
2	25	4
3	44	8
4	81	16
5	142	32
6	233	64
7	360	128
8	529	256
9	746	512
10	1017	1024

SO it looks like $n \ge 10$ works.

b)

FIRST DIRECTION:

Assume $n^3 + 17 \le 2^n$. We construct a formula on n variables that has exactly $n^3 + 17 \le 2^n$ satisfying assignments.

Let $\vec{b} \in \{T, F\}^n$. Let $\phi(\vec{b}, n)$ be the Boolean formula on $x_1, \ldots, x_n$ where which is satisfied exactly when the truth assignment is $\vec{b}$.

Examples:

$\vec{b} = (1, 0, 1)$ then

$\phi(\vec{b}, n)$ is $(x_1 \wedge \neg x_2 \wedge x_3)$

Let $\vec{b}_1$, $\vec{b}_2$, …, $\vec{b}_{n^3+17}$ be the first $n^3 + 17$ truth assignments. KEY: There are 2^n truth assignments so we needed to have $n^3 + 17 \le 2^n$ to have $\vec{b}_1$, $\vec{b}_2$, …, $\vec{b}_{n^3+17}$ be the

The formula is

$$\phi(\vec{b}_1, n) \wedge \cdots \wedge \phi(\vec{b}_{n^3+17})$$

SECOND DIRECTION:

Assume $n^3 + 17 > 2^n$. We must show that there is NO formula on n variables that has the EMILY property. If $\phi(x_1, \ldots, x_n)$ has the EMILY property then it has $n^3 + 17 > 2^n$ satisfying assignments. This is impossible since the max number of satisfying assignments is 2^n.

END OF SOLUTION

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5. (25 points) If x is a real number then $\lceil x \rceil$ is that number rounded DOWN. Examples: $\lfloor \pi \rfloor = 3$, $\lfloor \sqrt{2} \rfloor = 1$.

(a) (10 points) View the input x, y, z as the number in binary xyz which we denote (xyz). For example, 100 is 4.

Write a Truth Table for the following function with 3 inputs x, y, z and 3 outputs a, b, c.

$$f(x,y,z) = \begin{cases} \lfloor (xyz)^{1.5} - 1.7(xyz)) \rfloor & \text{if } \lfloor (xyz)^{1.5} - 1.7(xyz) \rfloor \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(EXAMPLE: Lets look at the row of the table that has $x = 1$,$y = 0$, $z = 1$. Since $(xyz) = (101) = 6$ and $\lfloor 6^{1.5} - (1.7) \times 6 \rfloor = \lfloor 4.49 \ldots \rfloor = 4$, the output is (100) so $a = 1$, $b = 0$, $c = 0$. Hence the below is part of the table.

x	y	z	a	b	c
1	0	1	1	0	0

SOLUTION

END OF SOLUTION

(b) (15 points) Convert your truth table into a formula. DO NOT SIMPLIFY.

SOLUTION

END OF SOLUTION

(c) (0 points- DO NOT HAND IN) Draw a circuit that computes that truth table.

SOLUTION

Solution Omitted.

END OF SOLUTION