Homework 2 MORALLY Due Feb 14 at 9:00AM
WARNING: THIS HW IS TWO PAGES LONG!!!!!!!!!!!!!!!!!

1. (25 points)

   (a) (10 points) Use truth table so show that

   $$\neg(p \vee q \vee r) \equiv \neg p \wedge \neg q \wedge \neg r$$

   (This is called *DeMorgan's law on three variables.*)

   (b) (15 points) Consider the statement:

   $$\text{For all } n\ [\neg(p_1 \vee \cdots \vee p_n) \equiv \neg p_1 \wedge \cdots \wedge \neg p_n]$$

   Prove it. Note that you cannot use Truth Table since we want it for all n. Do not use Induction (later when we learn induction we will do that). Use reasoning about what the truth table for both sides must look like.

GOTO NEXT PAGE

2. (25 points – 5 points each) For each of the following statements write the negation without using any negations signs.

   (a) $x = 4$

   (b) $x_1 \leq x_2 \leq x_3 \leq x_4$

   (c) $x \geq 5$ AND $x \geq 10$

GOTO NEXT PAGE

3. (35 points) Let $n \in \mathsf{N}$. NSQ(n) is the least number such that n can be written as the sum of NSQ(n) squares. Clearly NSQ(n) $\le n$ since

$$n = 1^2 + \cdots + 1^2.$$

It is known that NSQ(n) ≤ 4.

In this problem you will write and run two programs that will, given $n \in \mathsf{N}$, find a bound on NSQ(n).

(a) Write a program that will, given n, does the following:

- Find the largest n_1 such that $n_1^2 \le n$. If $n - n_1^2 = 0$ then you are done: $n = n_1^2$ so output 1. If not then goto the next step.
- Find the largest n_2 such that $n_2^2 \le n - n_1^2$. If $n - n_1^2 = 0$ then you are done: $n = n_1^2 + n_2^2$ so output 2. If not then ...
- Keep going like this until there is an output.

(b) Write a program that will, given n, find, for all $1 \le i \le n$, a number $A[i]$ such that i can be written as the sum of $A[i]$ squares.

- $A[0] \leftarrow 0$ (0 can be written as the sum of 0 squares).
- $A[1] \leftarrow 1$ (1 can be written as the sum of 1 square).
- For $i \leftarrow 2$ to n

$$A[i] \leftarrow 1 + \min\{A[i - j^2] \colon i - j^2 \ge 0\}$$

GOTO NEXT PAGE

4. (5 points) Run the first program on $n = 1, 2, \ldots, 1000$. List out all of the n where the answer was ≥ 5.

5. (0 points) How would you fill in the following sentence

   The first program on input n outputs a number ≥ 5 iff BLANK.

6. (5 points) Run the second program on $n = 1000$ (so we now know the answers for $1, \ldots, 1000$). List all of the n where $A[n]$ differs from the first program on n.

7. (0 points) How would you fill in the following sentence

   The first and second differ on n iff BLANK.

8. (5 points) List all of the n such that $A[n] = 4$.

9. (0 points) How would you fill in the following sentence

   $A[n] = 4$ *iff BLANK*

10. (Extra Credit) PROVE the following:

    There exists and infinite number of $x \in \mathsf{N}$ such that x cannot be written as the sum of ≤ 3 squares-of-naturals.