1. (0 points but please DO IT) What is your name?

2. (25 points) Let $a_n$ be defined by
\[
\begin{align*}
a_1 &= 1 \\
a_2 &= 21 \\
(\forall n \geq 3)[a_n &= 7a_{n-1} + 8a_{\left\lfloor \frac{n}{2} \right\rfloor} + 6].
\end{align*}
\]
Show by induction that, for all $n \geq 1$, $a_n \equiv 1 \pmod{20}$. 

3. (24 points) In class I prove the following for ways:

\[(\forall n \geq 3)(\exists d_1 < \cdots < d_n \in \mathbb{N})\left[1 = \frac{1}{d_1} + \cdots + \frac{1}{d_n}\right]\]

Each way I proved it can be re-interpreted as an algorithm that will, Given \( n \geq 3 \), generate \( d_1 < \cdots < d_n \) such that \( 1 = \frac{1}{d_1} + \cdots + \frac{1}{d_n} \).

(a) (8 points) Use the proofs to generate three ways to write 1 as the sum of FOUR reciprocals.

(b) (8 points) Use the proofs to generate four ways to write 1 as the sum of FIVE reciprocals.

(c) (8 points) Use the proofs to generate four ways to write 1 as the sum of SIX reciprocals.

DO NOT show work—just give us four \((d_1, d_2, d_3, d_4)\), four \((d_1, d_2, d_3, d_4, d_5)\), and four \((d_1, d_2, d_3, d_4, d_5, d_6)\).
4. (26 points) (As always, \( \mathbb{N} \) contains 0.) Bill has a statement \( P(n) \). He has proven the following:

\[
(\exists n \geq 100)[P(n)] \implies (\forall n \geq 10)[P(n)]
\]

Let

\[
A = \{n : P(n) \text{ is true} \}.
\]

(a) (13 points) List ALL possibilities for what \( A \) could be. (For a start: \( A \) could be empty, \( A \) could be \( \mathbb{N} \), \( A \) could be \( \{10, 11, 12, \ldots\} \), but there are more possibilities: Describe them all.)

(b) (13 points) How many possibilities are there for \( A \)? How many are finite? How many are infinite and have a finite complement? How many are infinite and have an infinite complement?
5. (25 points) In this problem we want you to write the answer in BOTH the notation form (e.g., \( \binom{11}{3} \)) and as an actual number (e.g., 165).

(a) (8 points) There are 10 people at a party. Everyone hugs everyone! How many hugs are there? (NOTE- this is after COVID. They are going to party like its 2019.)

(b) (8 points) There are 10 people at a party. Everyone gives everyone else a gift! How many gifts are there?

(c) (9 points) There are 20 students taking Ramsey Theory (all the cool kids are into the Ramsey!). They meet in a room like our classroom with tables. Bill demands that 6 sit at table 1, 5 sit at table 2, and 9 sit at table 3. How many ways can they do this (within a table we do not care where they sit).