Homework 7 MORALLY Due Apr 4 at 9:00AM
WARNING: THIS HW IS FOUR PAGES LONG!!!!!!!!!!!!!!!!!

1. (0 points but please DO IT) What is your name?

2. (25 points) Let a_n be defined by

   $a_1 = 1$

   $a_2 = 21$

   $(\forall n \geq 3)[a_n = 7a_{n-1} + 8a_{\lfloor \frac{n}{2} \rfloor} + 6]$.

   Show by induction that, for all $n \geq 1$, $a_n \equiv 1 \pmod{20}$.

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3. (24 points) In class I prove the following for ways:

$$(\forall n \geq 3)(\exists d_1 < \cdots < d_n \in \mathsf{N})\left[1 = \frac{1}{d_1} + \cdots + \frac{1}{d_n}\right]$$

Each way I proved it can be re-interpreted as an algorithm that will, Given $n \geq 3$, generate $d_1 < \cdots < d_n$ such that $1 = \frac{1}{d_1} + \cdots + \frac{1}{d_n}$.

(a) (8 points) Use the proofs to generate three ways to write 1 as the sum of FOUR reciprocals.

(b) (8 points) Use the proofs to generate four ways to write 1 as the sum of FIVE reciprocals.

(c) (8 points) Use the proofs to generate four ways to write 1 as the sum of SIX reciprocals.

DO NOT show work- just give us four (d_1, d_2, d_3, d_4), four $(d_1, d_2, d_3, d_4, d_5)$, and four $(d_1, d_2, d_3, d_4, d_5, d_6)$.

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4. (26 points) (As always, N contains 0.) Bill has a statement $P(n)$. He has proven the following:

$$(\exists n \geq 100)[P(n)] \implies (\forall n \geq 10)[P(n)]$$

Let

$$A = \{n \colon P(n) \text{ is true } \}.$$

(a) (13 points) List ALL possibilities for what A could be. (For a start: A could be empty, A could be N, A could be $\{10, 11, 12, \ldots\}$, but there are more possibilities: Describe them all.)

(b) (13 points) How many possibilities are there for A? How many are finite? How many are infinite and have a finite complement? How many are infinite and have an infinite complement?

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5. (25 points) In this problem we want you to write the answer in BOTH the notation form (e.g., $\binom{11}{3}$) and as an actual number (e.g., 165).

   (a) (8 points) There are 10 people at a party. Everyone hugs everyone! How many hugs are there? (NOTE- this is after COVID. They are going to party like its 2019.)

   (b) (8 points) There are 10 people at a party. Everyone gives everyone else a gift! How many gifts are there?

   (c) (9 points) There are 20 students taking Ramsey Theory (all the cool kids are into the Ramsey!). They meet in a room like our classroom with tables. Bill demands that 6 sit at table 1, 5 sit at table 2, and 9 sit at table 3. How many ways can they do this (within a table we do not care where they sit).