

NIM Problems for HW, Honors HW, Mid2untimed, Finaluntimed

WARNING: THIS HW IS FOUR PAGES LONG!!!!!!!!!!!!!!!!!!!!

Notation For this HW

- PI means Player I
- PII means Player II
- $\text{out}(x)$ means output x

1. (0 points but please DO IT) What is your name?
2. (X points) Consider the following NIM game: players can remove a SQUARE number of stones from the board. Let $W[n] =$
 - I if PI wins when the game starts with n stones;
 - II if PII wins when the game starts with n stones.

Write a program that will, given n , out $W[1], \dots, W[n]$.

Hint: Here is the psuedocode

$W[0] = II$

$W[1] = (I, 1)$

For $i = 2$ to n

if $(\exists x)[(0 \leq i - x^2 \leq n - 1) \wedge (W[i - x^2] = II)]$, out(I), else out(II).

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3. (X points) NIM with CASH is the following variant of NIM.

As with ordinary NIM, there is a set M of possible moves.

As with ordinary NIM, there is a set number of stones n that the board begins with.

UNLIKE ordinary NIM the players start out with money: PI starts with m_I , PII starts with m_{II} .

UNLIKE ordinary NIM, if a player removes x stones they must PAY x dollars.

If a player can't move, they lose; however, there are now 2 ways this can occur (a) there are 0 stones, or they have no money.

Let $W[n, m_I, m_{II}] =$

- I if PI wins $(n; m_I, m_{II})$ game.
- I if PII wins $(n; m_I, m_{II})$ game.

And NOW finally our problem. We take our game to be NIM with CASH with the set of moves $\{1, 3, 4\}$.

- (a) Write a program that will, given (n, m_I, m_{II}) determine who wins ALL of the following games:

$$\{(i, d_I, d_{II}) : (0 \leq i \leq n) \wedge (0 \leq d_I \leq m_I) \wedge (0 \leq d_{II} \leq m_{II})\}.$$

Hint: Here is the psuedocode

$$W[0, d_I, d_{II}] = II$$

$$W[i, 0, d_{II}] = II$$

For $i = 2$ to n

For $d_I = 1$ to n_I

For $d_{II} = 1$ to n_{II}

if $(\exists x)$

$$[(0 \leq i - x^2 \leq n - 1) \wedge (0 \leq x \leq d_I) \wedge (W[i - x^2, d_{II}, d_I - x] = II)]$$

then out(I), else out(II).

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4. (X points) Nim Misere is the variant where the person who picks up the last stone LOSES.
- (a) (0 points) Work out by (by hand or by code) who wins Nim Misere when $M = \{1, 2, 3\}$ for $n = 0$ to 20.
 - (b) (0 points) Make a conjecture of the form
PI wins Nim Misere with $M = \{1, 2, \}$ iff BLANK.
 - (c) (X points) Prove your conjecture.