

Homework X

Morally due Mon XX, 9:00AM

1. (X points) In this problem $\frac{n}{2}$ means $\lfloor \frac{n}{2} \rfloor$. In this problem we will be looking at the recurrence

$$a_1 = 1$$

$$(\forall n \geq 2)[a_n = a_{n-1} + a_{n/2}].$$

- (a) (0 points but you will need it for the later parts) Write a program that does the following:

On input d, N determine

For how many $1 \leq n \leq N$ is $a_n \equiv 0 \pmod{d}$.

For how many $1 \leq n \leq N$ is $a_n \equiv 1 \pmod{d}$.

For how many $1 \leq n \leq N$ is $a_n \equiv 2 \pmod{d}$.

\vdots

For how many $1 \leq n \leq N$ is $a_n \equiv N - 1 \pmod{d}$.

(Advice: Compute $a_n \pmod{N}$ instead of a_n to avoid large numbers.)

- (b) (10 points) Run your program for $N = 1000$ and $d = 2, 3, \dots, 20$. Present your data as follows (the numbers below are made up)

$$d = 2$$

c	$ \{n: n \equiv c \pmod{2}\} $
0	410
1	590

$$d = 3$$

c	$ \{n: n \equiv c \pmod{3}\} $
0	333
1	333
2	334

$$d = 4$$

c	$ \{n: n \equiv c \pmod{4}\} $
0	100
1	200
2	300
3	400

⋮

$d = 20$

c	$ \{n: n \equiv c \pmod{20}\} $
0	100
1	0
2	100
3	0
4	25
5	25
6	25
7	25
8	100
9	0
10	100
11	0
12	25
13	25
14	25
15	25
16	100
17	100
18	200
19	0

- (c) (X points) Based on your data make a conjecture of the form:
Let c, d be such that $0 \leq c \leq d - 1$ and $d \geq 2$. There exists an infinite number of n such that $a_n \equiv c \pmod{d}$ IFF $XXX(c, d)$.
- (d) (X points) Based on your data make a conjecture of the form:
Let c, d be such that $0 \leq c \leq d - 1$ and $d \geq 2$. For large N the set

$$\{n: a_n \equiv c \pmod{d}\} \cap \{1, \dots, N\}$$

is ROUGHLY size $\frac{N}{c}$ iff $YYY(c, d)$.