Homework 2 Programming Part

Morally due Mon Feb 14, 9:00AM

1. (55 points) Let $n \in \mathsf{N}$. NSQ(n) is the least number such that n can be written as the sum of NSQ(n) squares. Clearly NSQ(n) $\le n$ since

$$n = 1^2 + \cdots + 1^2.$$

It is known that NSQ(n) ≤ 4.

In this problem you will write and run two programs that will, given $n \in \mathsf{N}$, find a bound on NSQ(n).

(a) Write a program that will, given n, does the following:

- Find the largest n_1 such that $n_1^2 \le n$. If $n - n_1^2 = 0$ then you are done: $n = n_1^2$ so output 1. If not then goto the next step.
- Find the largest n_2 such that $n_2^2 \le n - n_1^2$. If $n - n_1^2 = 0$ then you are done: $n = n_1^2 + n_2^2$ so output 2. If not then ...
- Keep going like this until there is an output.

(b) Write a program that will, given n, find, for all $1 \le i \le n$, a number $A[i]$ such that i can be written as the sum of $A[i]$ squares.

- $A[0] \leftarrow 0$ (0 can be written as the sum of 0 squares).
- $A[1] \leftarrow 1$ (1 can be written as the sum of 1 square).
- For $i \leftarrow 2$ to n

$$A[i] \leftarrow 1 + \min\{A[i - j^2] \colon i - j^2 \ge 0\}$$

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2. (15 points) Run the first program on $n = 1, 2, \ldots, 1000$. List out all of the n where the answer was ≥ 5.

3. (0 points) How would you fill in the following sentence

   The first program on input n outputs a number ≥ 5 iff BLANK.

4. (15 points) Run the second program on $n = 1000$ (so we now know the answers for $1, \ldots, 1000$). List all of the n where $A[n]$ differs from the first program on n.

5. (0 points) How would you fill in the following sentence

   The first and second differ on n iff BLANK.

6. (15 points) List all of the n such that $A[n] = 4$.

7. (0 points) How would you fill in the following sentence

   $A[n] = 4$ *iff BLANK*

8. (Extra Credit) PROVE the following:

   There exists and infinite number of $x \in \mathsf{N}$ such that x cannot be written as the sum of ≤ 3 squares-of-naturals.