1. (15 points) Let $p$ and $q$ be primes. Let $n = p^2q^3$. Show that, $n^{2/5} \notin \mathbb{Q}$. Use Unique Factorization.
2. (20 points)

(a) (7 points-1 point each) Fill in the following:
   0) \( 0^4 \equiv \) (mod 8).
   1) \( 1^4 \equiv \) (mod 8).
   2) \( 2^4 \equiv \) (mod 8).
   3) \( 3^4 \equiv \) (mod 8).
   4) \( 4^4 \equiv \) (mod 8).
   5) \( 5^4 \equiv \) (mod 8).
   6) \( 6^4 \equiv \) (mod 8).
   7) \( 7^4 \equiv \) (mod 8).

(b) (13 points) Show that there exists an infinite number of \( n \) such that \( n \) cannot be written as the sum of 6 fourth powers. (HINT: Use Part a.)
3. (15 points) Find a number $M$ such that the following is true, and prove it.

$$(\forall n \geq M)(\exists x, y \in \mathbb{N})[n = 37x + 38y].$$