Combinatorial Identities

250H

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Proof (2): Consider the identity, $(x + y)^n = \sum \binom{n}{i} x^i y^{n-i}$

Choose $x = y = 1$. Now we have $(1 + 1)^n = \sum \binom{n}{i} 1^i 1^{n-i}$ or $2^n = \sum \binom{n}{i}$.

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So we have $\binom{2n}{n}$ ways.

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Option 1: Lets ignore gender. Then we have a total of 2*n* people.

So we have $\binom{2n}{n}$ ways.

Option 2: We can pick 0 girls and *n* boys, 1 girl and *n*-1 boys, ..., *n* girls and *n*-1 boys.

So we have $\sum \binom{n}{i}^2$ ways.

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Option 2: We can pick 0 girls and *n* boys, 1 girl and *n*-1 boys, ..., *n* girls and *n*-1 boys.

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This is another identity: $\sum \binom{n}{i}^2 = \binom{2n}{n}$

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1. $(x+y)^n = \sum \binom{n}{i} x^i y^{n-i}$

2. $\sum \binom{n}{i}^2 = \binom{2n}{n}$