Combinatorics and Probability Problems

250H

Counting

How many different passwords of 4 uppercase letters followed by 2 digits with none of the letters repeated can people have? (Note: The digits can repeat)

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$$4 \cdot 4 \cdot 4 \cdot 4 \cdot 1 \cdot 1 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 65536$$

Binomial Coefficients

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By the Binomial Theorem, we have $(-x+3y)^{11} = \sum_{j=0}^{11} \binom{11}{j} (-x)^{11-j}(3y)^j$

We plug in j = 6

$$\binom{11}{6}(-x)^{11-6}(3y)^6$$
$$\binom{11}{6}(-x)^5(3y)^6$$
$$462(-1^5)(3^6)x^5y^6$$
$$462(-1)(729)x^5y^6$$
$$-336798x^5y^6$$

So our coefficient is -336798.

Permutations and Combinations

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Since order doesn't matter and repetition is allowed, we can use the stars and bars formula. Here are bins are the 20 colors and the stars are 15-3 = 12 as 3 must be black. So,

$$C(20+12-1,12) = C(31,12) = \frac{31!}{12!(19)!}$$

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blueberry has 9 letters, with 2 b's, 1 l, 1 u, 2 e's, 2 r's, and 1 y. So,

$$\frac{9!}{2!2!2!} = 45360$$

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$$\frac{C(4,1)C(13,5)}{C(52,5)} = \frac{4(1287)}{2598960} = \frac{5148}{2598960} = \frac{33}{16660}$$

Probability

Suppose that Alice selects a mushroom by first picking one of two gardens at random and then selecting a mushroom from this garden. The first garden contains three red mushrooms and four black mushrooms, and the second garden contains five red mushrooms and six black mushrooms. Use Bayes' theorem to find the probability that Alice picked a mushroom from the second garden if she has selected a red mushroom.

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Let E be the red mushroom.

Let F be the mushroom picked from the second garden.

$$P(F) = P(\overline{F}) = \frac{1}{2}$$

$$P(E \mid F) = \frac{5}{5+6} = \frac{5}{11}$$

$$P(E \mid \overline{F}) = \frac{3}{3+4} = \frac{3}{7}$$

$$\begin{aligned} P(F \mid E) &= \frac{P(E \mid F)P(F)}{P(E \mid F)P(F) + P(E \mid \overline{F})P(\overline{F})} \\ &= \frac{\frac{5}{11}\left(\frac{1}{2}\right)}{\frac{5}{11}\left(\frac{1}{2}\right) + \frac{3}{7}\left(\frac{1}{2}\right)} \\ &= \frac{\frac{5}{22}}{\frac{5}{22} + \frac{3}{14}} \\ &= \frac{35}{68} \end{aligned}$$