

Extra Problems Similar to H01

1. For a variety of x and y of your choice: Give a Propositional Formula on x variables that has exactly y satisfying assignments. For which x, y can you do this?
2. Write down two 3-var formulas and do truth table for them. If they have the same truth table, then great, they are the same. If not then INDICATE which rows they differ on.
3. (a) Fill in the $X(n)$ and $Y(n)$. If ϕ has n variables then the number of satisfying assignments is between $X(n)$ and $Y(n)$.
(b) Let $Z(n)$ be the midpoint of $X(n)$ and $Y(n)$ (round down if needed). Describe how you would construct a formula on n variables with $Z(n)$ satisfying assignments.
4. (30 points) (NOTE: 0 and 1 are NOT prime. You will need that for this problem.)
(a) (15 points) View the input x, y, z as the number in binary xyz which we denote (xyz) . For example, 100 is 4.
Write a Truth Table for the following function with 3 inputs x, y, z and 1 output a .

$$f(x, y, z) = \begin{cases} 0 & \text{if } (xyz) \text{ is NOT A SQUARE.} \\ 1 & \text{if } (xyz) \text{ is SQUARE.} \end{cases}$$

- (b) Write the formula.
- (c) Draw the Circuit
- (d) Obtain a formula and circuit for this problem on 4-bit numbers.
ADVICE: DO NOT write the truth table, JUST write the entries of the truth table where the output is 1. Then find the formula.