

Honors Homework 2

Morally Due Mon Feb 19 at 10:00AM

1. (0 points) What is your name? Write it clearly.

2. (30 points) Consider the DUP-SPOILER game with $(\mathbb{Q}, \mathbb{R}, 4)$. So the orderings are \mathbb{Q} , the rationals, \mathbb{R} the reals and the game will go for 3 rounds. We denote DUP by D and SPOILER by S.

(a) Assume the game goes as follows:

- i. Round 1: S picks $\sqrt{2}$ in \mathbb{R} . D picks 0 in \mathbb{Q} .
- ii. Round 2: S picks $\frac{1}{1000}$ in \mathbb{Q} . D picks 2 in \mathbb{R} .
- iii. Round 3: S picks $\frac{1}{2000}$ in \mathbb{Q} .

Give a move D can make to WIN!

(b) Assume the game goes as follows:

- i. Round 1: S picks $\sqrt{2}$ in \mathbb{R} . D picks 0 in \mathbb{Q} .
- ii. Round 2: S picks $\frac{1}{1000}$ in \mathbb{Q} . D picks 2 in \mathbb{R} .
- iii. Round 3: S picks $\sqrt{5}$ in \mathbb{R} .

Give a move D can make to WIN!

3. (30 points) Consider the DUP-SPOILER game with $(\mathbb{Q}, \mathbb{R}, 1000)$. So the orderings are \mathbb{Q} , the rationals, \mathbb{R} the reals and the game will go for 1000 rounds. We denote DUP by D and SPOILER by S.

Assume that after 999 moves:

- The points in \mathbb{R} picked are $r_1 < r_2 < \dots < r_{999}$. (They did not have to be picked in that order. For example, S's first move could be what we now call r_{10} .)
 - The points in \mathbb{Q} picked are $q_1 < q_2 < \dots < q_{999}$. (They did not have to be picked in that order. For example, S's first move could be what we now call q_{10} .)
 - If in round j S picked r_i then in that round D picked q_i .
 - If in round j S picked q_i then in that round D picked r_i .
- (a) Assume that in Round 1000 S picks a point $r \in \mathbb{R}$ such that $r < r_1$. How should D respond to win? (The kind of answer I want, and this is NOT correct, is *S picks a point between q_{10} and q_{20}* .)
- (b) Assume that in Round 1000 S picks a point $q \in \mathbb{Q}$ such that $q_{10} < q < q_{11}$. How should D respond to win?
- (c) Assume that in Round 1000 S picks a point r such that $r_{999} < r$. How should D respond to win?

4. (40 points) Informally argue that, for all n , D wins the game (Q, R, n) .