Homework 3, MORALLY Due Feb 19

1. (30 points–6 points each) In this problem the domain is N.

   (a) Express the following statements using quantifiers. There exist an (x, y, z), with $x, y, z \geq 2$ all distinct, such that $x^2 + y^2 = z^2$.

   (b) Express the following statements using quantifies. There exist an INFINITE NUMBER of (x, y, z), with $x, y, z \geq 2$ and all distinct, such that $x^2 + y^2 = z^2$. (This happens to be TRUE but you do not need that for this problem.)

   (c) Express the following statements using quantifies. There is NO x, y, z, n with $x, y, z \geq 2$ and $n \geq 3$ such that $x^n + y^n = z^n$.

   (d) The statement in Part d is TRUE. It is called *Fermat's Last Theorem.* Look it up and write a paragraph about it including who proved it, when, and how long it was open for.

2. (30 points–15 points each) In this problem the domain is R.

   (a) Express the following statements using quantifies. Every polynomial with real coefficients, of degree 3, has a real solution.

   (b) The statement in Part a is TRUE! Give a proof (it can be informal, this is NOT Math 410).

3. (40 points–8 points each)

   For this problem we use the following standard terminology:

   A domain is *dense* if $(\forall x, y)[x < y \Rightarrow (\exists z)[x < z < y]]$.

   A domain has a *min element* if $(\exists x)(\forall y)[x \le y]$.

   A domain has a *max element* if $(\exists x)(\forall y)[y \le x]$.

   In this problem we list conditions on a domain. EITHER give a domain that satisfies the conditions OR state that there is NO such domain (no proof required).

   (a) D is finite and dense.

   (b) D is finite and dense and has ≥ 2 elements.

   (c) D is infinite and not dense

   (d) D is infinite, has a min element, has a max element, and is dense.

   (e) D is infinite, has a min element, has a max element, and is NOT dense.