Homework 5, MORALLY Due March 4

1. (25 points) Let p be a prime. Show that $\sqrt{p} \notin \mathsf{Q}$ using Unique Factorization.

2. (25 points) Let $c \in \mathsf{N}$ with $c \geq 2$. Let p be a prime. Show that $p^{1/c} \notin \mathsf{Q}$.

3. (25 points)

   (a) (0 points but you'll need it) Write a program that will, given n, tell if n is prime. (If this is a library in Python, thats fine.)

   (b) (0 points but you'll need it) Write a program that will, given n, return the NUMBER OF PRIMES $\le n$. We call this $\pi(n)$. (This has nothing to do with π but its traditional.)

   (c) (0 points but you'll need it) Write a program that will, given N and L produce a table of $\pi(L)$, $\pi(2L)$, ..., $\pi(LN)$. For example, if $N = 10$ and $L = 4$ then the output is

x	$\pi(x)$
4	2
8	4
12	5
16	6
20	8
24	9
28	9
32	11
36	11
40	12

   (d) (0 points but you'll need it) Write a program that will, given N and L produce a table of $\pi(L)/L$, $\pi(2L)/2L$, ..., $\pi(LN)/LN$. For example, if $N = 10$ and $L = 4$ then the output is

x	$\pi(x)/x$
4	0.5
8	0.5
12	0.42
16	0.38
20	0.4
24	0.38
28	0.32
32	0.34
36	0.31
40	0.3

(e) (25 points) Run the program in the last problem on $N = 10,000$ and $L = 10$. Plot it. Optional: See if you can find an equation that approximates it.

4. Let

$$D = \{4n + 1 : n \in \mathsf{N}\}.$$

We list out the first few elements and note if they are primes, units, or composites IN D.

$4n+1$	status	factorization if composite
1	unit	
5	prime	
9	prime, really!	
13	prime	
17	prime	
21	prime, really!	
25	comp, finally!	5×5
29	prime	
33	prime, really!	
37	prime	
41	prime	
45	comp	5×9

Gee, there *seem* to be lots more primes in D then in N. But is this true for large N? Yes, but HOW true is it?

(a) (0 points but you'll need it) Write a program that will, given x, tell if n is prime IN D. (NOTE- the program must also test if $x \in D$.)

(b) (0 points but you'll need it) Write a program that will, given n, return the NUMBER OF PRIMES IN D that are $\leq n$. We call this $\pi_D(n)$.

x	$\pi_D(x)$
4	0
8	1
12	2
16	3
20	4
24	5
28	5
32	6
36	7
40	8

(c) (0 points but you'll need it) Write a program that will, given N and L produce a table of $\pi(L)/(L/4)$, $\pi(2L)/(2L/4)$, ..., $\pi(LN)/(LN/4)$. (We divide by $kL/4$ instead of of just by kL since the number of elements of D that are $\leq L$ is roughly $L/4$. For example, if $N = 10$ and $L = 4$ then the output is

x	$\pi_D(x)/(x/4)$
4	0
8	0.5
12	0.66
16	0.75
20	0.8
24	0.83
28	0.71
32	0.75
36	0.77
40	0.8

(d) (25 points) Run the program in the last problem on $N = 10,000$ and $L = 10$. Plot it. Optional: See if you can find an equation that approximates it.