Homework 8, MORALLY Due April 1

1. (30 points) Recall what induction is:

   From

   - $P(0)$
   - $(\forall n \geq 0)[P(n) \Rightarrow P(n+1)]$

   You have

   $(\forall n)[P(n)]$.

   Induction is great for proving theorems of the form $(\forall n \in \mathsf{N})[P(n)]$.

   We want to prove theorems of the form

   $$(\forall z \in \mathsf{Z})[P(z)].$$

   Give a scheme similar to the one above that will have the conclusion $(\forall z \in \mathsf{Z})[P(z)]$.

2. (35 points) Consider the recurrence

   $T(1) = 10$

   $T(2) = 30.$

$$(\forall n \geq 3)\left[T(n) = T\left(\left\lfloor \frac{n}{a} \right\rfloor\right) + T\left(\left\lfloor \frac{n}{b} \right\rfloor\right) + cn\right].$$

   Find an infinite number of triples (a, b, c) of POSITIVE rationals such that

$$(\forall n \geq 1)[T(n) \leq 100n].$$

3. (35 point) Consider the following recurrence:

   $T(0) = 1$

   $T(1) = 6$

   $T(2) = 21$

   $(\forall n \geq 3)[T(n) = 2T(n-1) + 4T(\lfloor \sqrt{n} \rfloor) + 5n$.

   Show that, for all $n \geq 1$, $T(n) \equiv 1 \pmod 5$.

   Hint: Use Strong Induction.