Homework 9, MORALLY Due 10:00AM April 22

1. (30 points) Recall the BEE sequence.

   $a_1 = 1$

   $(\forall n \geq 2)[a_n = a_{n-1} + a_{\lfloor n/2 \rfloor}]$

   Try to prove the following:

   There are an infinite number of n such that $a_n \equiv 0 \pmod{9}$.

   You will do this by following the proof for moe 7. Show where the proof breaks down.

   (Recall that the the statement $(\exists^\infty n)[a_n \equiv 0 \pmod{9}]$ is not known to be true or false, though empirical evidece suggests that its true. Our approach did not work; however, some other approach might.)

2. (30 points) Recall that $|\mathrm{SPS}(1,\ldots,n)| = \frac{n(n+1)}{2} + 1$.
   Assume n is large. What is $|\mathrm{SPS}(1,\ldots,n-1,2^n)|$?

3. (40 points) Recall that the AM-GM inequality is

   For all $n \geq 2$, for all $x_1, \ldots, x_n \in \mathsf{R}^+$

$$\frac{x_1 + \cdots + x_n}{n} \geq (x_1 \cdots x_n)^{1/n}.$$

   with equality iff $x_1 = \cdots = x_n$.

   We are wondering: When are the Arithmetic Mean and the Geometric Mean the furthest apart?

   (a) Write a program that will do the following:

      i. Input n, N
      ii. For all subsets $\{x_1 < \cdots < x_n\}$ of $\{1, \ldots, N\}$ of size n compute $A = \frac{x_1 + \cdots + x_n}{n}$, $G = (x_1 \cdots x_n)^{1/n}$, $D = A - G$
      iii. Then print out the top n values of D and which $x_1 < \cdots < x_n$ lead to them.
      iv. **Send Code to Emily: ekaplitz@umd.edu**

   (b) (30 points) Run the program for $(n, N) = (5, 10)$ and present the results.

   (c) (10 points) Run the program for $1 \leq n \leq 20$ and $n \leq N \leq 20$. Make a conjecture about what types of $x_1 < \cdots < x_n$ lead to large values of D.