Homework 10, MORALLY Due 10:00AM April 29

1. (25 points) Bill makes his Gnilrad lunch that consists of the following:

   - Sandwich: Egg Salad OR Tuna Fish OR Cheese.
   - Fruit: Apple OR Orange OR Grapes OR Blueberries OR Strawberries.
   - Desert: Apple Sauce OR Cookie.

   For the questions below show your work but also give us an actual number like 10 and not just the notation like $\binom{5}{3}$.

   (a) (5 points) How many ways can Bill make lunch for his Gnilrad? NOTE: She has ONE sandwitch, ONE fruit and ONE desert.

   (b) (10 points) One day she complains: *I don't want to have my Fruit be an apple, and my desert be Applesauce at the same time, though I am okay with having one or the other.* Bill obeys her wishes. NOW how many ways can Bill make lunch for her? NOTE: Examples: Darling is happy with Eggsalad-Apple-Cookie but NOT with Eggssalad-Apple-Applesauce.)

   (c) (10 points) One day she complains: *(1) I don't want to have my an apple and applesauce at the same time, AND (2) I want 2 different Sandwiches AND (3) I want 3 different Fruits AND (4) I still just want one Desert.* Bill obeys her wishes. NOW how many ways can Bill make lunch for her?

   (d) (0 points but you must answer it) Why does Bill call her *Gnilrad*?

(25 points) On the last slide of the lecture *The Law of Inclusion and Exclusion* is the law for A_1, A_2, A_3, A_4. Its complicated!

(a) (10 points) What if the following hold

- Each A_i has x_1 elements.
- Each intersection of TWO sets has x_2 elements.
- Each intersection of THREE sets has x_3 elements.
- Each intersection of FOUR sets has x_4 elements.

Give an expression for $|A_1 \cup A_2 \cup A_3 \cup A_4|$ in terms of x_1, x_2, x_3, x_4. It should be much simpler then the general law.

(b) (15 points) Let $A_1, \ldots, A_n$ be sets. Assume that, for $1 \le i \le n$, the intersection of i of these sets has size x_i. Give an expression for $|A_1 \cup \cdots \cup A_n|$ in terms of $x_1, \ldots, x_n$. You CANNOT use DOT DOT DOT. You can and should use a summation sign.

2. (25 points) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 100$$

with $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, and $x_4 \geq 4$.

3. (25 points) Read or re-read the slides on the Horse Numbers. The numbers $H(n)$ will come up in this problem.

   For $n \geq 2$. Let $I(n)$ be the number of ways that n horses, $x_1, \ldots, x_n$, can finish a race (so orderings with equalities allowed) that have $x_1 < x_n$.

   (a) (0 points but you should do it or convince yourself that you could). What is $I(2)$, $I(3)$, $I(4)$. Do $I(4)$ in such a way that it can be generalized to $I(n)$.

   (b) (25 points) Give a recurrence for $I(n)$. It may also involve $H(n)$. For example, it could be (but its NOT) $I(n) = I(n-1) + I(n-4) + H(n)H(n-3)$.