Homework 11, MORALLY Due 10:00AM May 6

1. (30 points- 5 points each) After Emily gets her PhD she works for a casino. She has the following dyslexic idea: Rather than have 13 ranks and 4 suites, lets have 13 suites and 4 ranks! She also changes the number of cards in a hand to 4 so that a straight is possible.

   (a) How many poker hands are there?

   (b) What is the probability of getting a straight flush?

   (c) What is the probability of getting a straight that is not a straight flush?

   (d) What is the probability of getting a flush that is not a straight flush?

   (e) What is the probability of getting 4-of-a-kind?

   (f) What is the probability of getting 3-of-a-kind that is NOT 4-of-a-kind?

2. (25 points) Bill is looking at three dice. Bill will pick up one die and roll it.

- Dice 1 has 3 sides labelled 1,2,3. The probabilities are:
  prob(1)=0.3, prob(2)=0.3, prob(3)=0.4.
  Bill picks up this die with probability 0.5
- Dice 2 has 4 sides labelled 1,2,3,4. The probabilities are:
  prob(1)=0.3, prob(2)=0.3, prob(3)=0.2, prob(4)=0.2.
  Bill picks up this die with probability 0.3
- Dice 3 has 5 sides labelled 1,2,3,4,5. The probabilities are:
  prob(1)=prob(2)=prob(3)=prob(4)=prob(5)=0.2.
  Bill picks up this die with probability 0.2

(a) (10 points) What is the probability that Bill rolls a 3? Show your work.

(b) (15 points) (You will probably want to write a program for this one.) Give a table like the one below (the numbers in it are WRONG, yours should be right).

Number	Prob
1	0.3
2	0.2
3	0.1
4	0.2
5	0.2

3. (20 points) (This problem was inspired by a comment Soren made.)

   Let $E(k,n)$ be the number of solutions in N to

$$x_1 + \cdots + x_k = n.$$

   In class we showed that $E(k,n) = \binom{n+k-1}{n}$.

   (a) (10 points) Pretend that you *do not know* the formula for $E(k,n)$. But you (actually Soren) have the following idea!

   EITHER $x_k = 0$ OR $x_k = 1$ OR $x_k = 2$ OR $\cdots$ $x_k = n$.

   Use this to get a recurrence for $E(k,n)$. Also make sure to have a base case (perhaps more than one) that makes sense.

   (b) (10 points) From Part 1 you have a recurrence for $E(k,n)$. Now use the fact that you DO know $E(k,n) = \binom{n+k-1}{n}$ to get a combinatorial identity.

4. (25 points) Fill in the $f(c)$ below and then prove your statement:

   For any c-coloring of the $(c+1) \times f(c)$ grid there is a monochromatic rectangle.