250 Midterm March 31, 2026 9:30AM-10:45AM

1. (0 points) What is your name?

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2. (20 points) This is a FILL IN THE BLANK question. No proof needed. Find $X \in \mathsf{N}$ such that the following is true:

   For all $n \geq X$, there is a boolean formula on n variables that is satisfied by exactly $n^2 + n$ satisfying assignments.

   (**YOU DO NOT HAVE TO GIVE US THE FORMULA**)

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3. (20 points) Use unique factorization to show the following:

   Let $n \in \mathsf{N}$*, and* $n \geq 2$*. Let* $n = p_1^{n_1} \cdots p_k^{n_k}$*, where* $p_1, \ldots, p_k$ *are primes. If* $n^{1/3}$ *is rational then* $(\forall 1 \leq i \leq k)[n_i \equiv 0 \pmod 3]$.

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4. (20 points)

(a) Compute the following mod 9:

$$0^6, 1^6, 2^6, \ldots, 8^6.$$

(You may use the following shortcut: $(9-a)^6 \equiv a^6 \pmod 9$.)

(b) Use the results of part a to find a number N and an infinite set X such that the following is true

If $x \in X$, then x cannot be written as the sum of N sixth powers.

Make N as large as possible.

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5. (20 points) Let a_n be defined as follows

$a_0 = 11$
$a_1 = 21$
$(\forall n \geq 2)[a_n = 7a_{n-1} + 4a_{n-2}]$

Prove that
$(\forall n \geq 0)[a_n \equiv 1 \pmod{10}]$.

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6. (20 points) In this problem (1) all symbos have their usual meaning, and (2) a domain is a subset of R.

   For this problem NO proofs are required; however, if you give a domain it has to be correct, and if you say there is no such domain, that has to be correct.

   Consider the following sentence:

$$(A) \qquad (\forall x)(\exists y)[x + y = \sqrt{2}].$$

   (a) (10 points) Do one of the following:

   - Give an infinite domain where A is true.
   - State that there is no such domain.

   (b) (10 points) Do one of the following:

   - Give a finite domain with at least 10 elements where A is true.
   - State that there is no such domain.

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