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In any SAT assignment need $x_1 = T$ and $x_3 = F$ so $\neg x_1 \vee x_3$ is F.
Hence NOT in SAT.

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YES If ϕ is in 3-CNF form (we'll define that later) then there exists a randomized 1.306^n algorithm.

UNKNOWN TO SCIENCE If there are no restrictions on the formula, then unknown if there is an algorithm better than $\sim 2^n$.

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Notation We denote Polynomial Time by **P**.

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UNKNOWN TO SCIENCE In fact, The $(1.306)^n$ algorithm is the best algorithm we know.

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How Long Has It Been Open? Posed in 1971 by Stephen Cook and Leonid Levin independently. So around 50 years.

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There are **thousands** of problems are equiv to SAT. Hence:

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1. **Traveling Salesperson Problem (TSP)** Given n cities and how much it costs to go from any city to an city, determine cheapest way to visit all cities. Studied since the 1930's.
2. **Scheduling** Given n rooms and when they are free, and given m people who are requesting them for certain timeslots, can you accommodates all of them? Studied since the 1880's.

The following is known:
(3-SAT is in P) $\leftrightarrow$ (TSP is in P) $\leftrightarrow$ (SCHED is in P).
There are **thousands** of problems are equiv to SAT. Hence:

- The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.

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- The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.
- Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.

Proper Terminology and What Do People In the Know Think?

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If you want to see the poll, here is the link:

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Some Good News

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More generally, if you know a problem is equivalent to SAT then you know that you should not look for an optimal poly time solutions. There are many other options to try.

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Jace can go to 8196, which is further than I can go.

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Bill Leonid Levin was in the USSR, Stephen Cook was in America, and in those days communication between the two was very hard. But you raise an interesting point. (Next slide).

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