Intermediary Truth Values

TRUE is 1, FALSE is 0

We have dealt with **math** where statement really are **true** or **false** (with some rare exceptions).

The **Real World** is messier!

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- Joe Biden was a good president. I won't even try to give this one a number because it's to ill defined. (Also I should not share my politics with the class.)
- Its going to rain tomorrow. Forecasts give probabilities.

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We will **not** dwell on this. We will ponder a well defined math question about intermediary truth values.

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1. When $x, y \in \{0, 1\}$ should give the same answer as usual case.
2. Want the definitions to satisfy De Morgan' Law.
3. Want the definitions to make sense intuitively. For example,
   $x \wedge y \leq x$ (harder for $x \wedge y$ to be true then for x to be true)
   $x \vee y \geq x$ (easier for $x \vee y$ to be true then for x to be true)

Work on in groups!

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Answer on the next page.

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