Simplifying Mathematical Expressions

The $\wedge$ of Two $\leq$ Statements

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The $\wedge$ of a $\leq$ and a $\geq$

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Yes:

$$3 \leq x \leq 10$$

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$$(x \leq A_1) \wedge \cdots \wedge (x \leq A_n) \wedge (x \geq B_1) \wedge \cdots \wedge (x \geq B_m)$$

$$\equiv (B_m \leq x \leq A_1)$$

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DeMorgan's law **does** help us:

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Can always get rid of $\neg$. We omit generalizations though they may be on the HW.