Optional Project Solutions

May 7, 2026

Problem 2a: Prop Logic

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Give a formula on 5 variables that has exactly 3 satisfying assignments.

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Imagine a truth table on 5 vars which the first three rows are T but the rest are F. The formula you get is

$$(\neg v \wedge \neg w \wedge \neg x \wedge \neg y \wedge \neg z)$$

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Problem 2b: Prop Logic

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Give a formula on 3 variables that has exactly 5 satisfying assignments.

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Problem 2c: Prop Logic

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Give an a, b such that there is NO formula on a variables that has exactly b satisfying assignments.

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Give an a, b such that there is NO formula on a variables that has exactly b satisfying assignments.

Take $a = 3$ and $b = 2^a + 1 = 10$.

Problem 2d: Prop Logic

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You would need both $b \geq 2^a + 1$ and $a \geq 2^b + 1$.

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You would need both $b \geq 2^a + 1$ and $a \geq 2^b + 1$.
If both are true then

$$b \geq 2^a + 1 \geq 2^{2^b+1} + 1$$

which is impossible.

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$(\forall x \neq H)(\exists y)x < y \wedge (\forall z)[\neg(x < z < y)]$.
(For every element $x \neq H$ there is a *successor*.)

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$(\forall x \neq H)(\exists y)x < y \wedge (\forall z)[\neg(x < z < y)]$.
(For every element $x \neq H$ there is a *successor*.)

$(\forall x \neq L)(\exists y)y < x \wedge (\forall z)[\neg(y < z < z)]$.
(For every element $x \neq L$ there is a *predecessor*.)

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Let D be the set

$$\left\{-1, -\frac{1}{2}, -\frac{1}{3}, \dots\right\} \cup \left\{\dots \frac{1}{3}, \frac{1}{2}, 1\right\}.$$

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Every element except for -1 and 1 has a successor and a predecessor respectively.

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Use const ind to find b, m such that

for all n, $a_n \equiv b \pmod{m}$.

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So YES, $b = 17$ and $m = 23$.

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Next slide for exciting conclusion.

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$b^7 p_1^{n_1} \cdots p_k^{n_k} = a^7.$

$a = p_1^{a_1} \cdots p_k^{a_k}$ hence $a^7 = p_1^{7a_1} \cdots p_k^{7a_k}.$

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$$(p_1^{7b_1} \cdots p_k^{ba_k})(p_1^{n_1} \cdots p_k^{n_k}) = a^7 = p_1^{7a_1} \cdots p_k^{7a_k}.$$

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$$p_1^{7b_1+n_1} \cdots p_k^{7b_k+n_k} = p_1^{7a_1} \cdots p_k^{7a_k}$$

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$$p_1^{7b_1+n_1} \cdots p_k^{7b_k+n_k} = p_1^{7a_1} \cdots p_k^{7a_k}$$

Let $1 \leq i \leq k$. By Unique Factorization, p_i has to appear the same number of times on both sides.

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$$(p_1^{7b_1} \cdots p_k^{ba_k})(p_1^{n_1} \cdots p_k^{n_k}) = a^7 = p_1^{7a_1} \cdots p_k^{7a_k}.$$

$$p_1^{7b_1+n_1} \cdots p_k^{7b_k+n_k} = p_1^{7a_1} \cdots p_k^{7a_k}$$

Let $1 \le i \le k$. By Unique Factorization, p_i has to appear the same number of times on both sides.

Hence

$7b_i + n_i = 7a_i$

$n_i = 7(a_i - b_i) \equiv 0 \pmod 7.$

Darling's Lunch: Combinatorics

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Bill makes his Darling lunch.

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S sandwiches, F fruits, D desserts.

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How many ways can Darling have lunch?
Sandwiches: $\binom{S}{s}$ Fruits: $\binom{F}{f}$
Desserts: For $1 \le i \le D$ Darling will get x_i of dessert i. So the number of ways she can get deserts is the number of solutions to $x_1 + \cdots + x_D = d$.

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Final answer: $\binom{S}{s}\binom{F}{f}\binom{D+d-1}{(D-1)!d!}$.

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PBJ, BLUEBERRY, BLUEBERRY PIE: Darling does not want ALL THREE.

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How many ways can Darling have lunch?

How many lunches **do** have all 3?: $\binom{S-1}{s-1}\binom{F-1}{f-1}\binom{D+d-2}{(D-2)!(d-1)!}$.

So answer is $\binom{S}{s}\binom{F}{f}\binom{D+d-1}{(D-1)!d!} - \binom{S-1}{s-1}\binom{F-1}{f-1}\binom{D+d-2}{(D-2)!(d-1)!}$.

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and $x_1 \geq 1$, $x_2 \geq 2$, ,$x_3 \geq 3$, $x_4 \geq 4$, and $x_5 \geq 5$.

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Stars and Bars!

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Four Bars.

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1st space has at least 1.

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Stars and Bars!
Four Bars.
1st space has at least 1. 2nd space has at least 2.

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3rd space has at least 3.

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3rd space has at least 3. 4th space has at least 4

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Stars and Bars!

Four Bars.

1st space has at least 1. 2nd space has at least 2.

3rd space has at least 3. 4th space has at least 4

5th space has at least 5

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So $1 + 2 + 3 + 4 + 5 = 15$ stars already placed.

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Our problem: How many ways to arrange 4 bars and
$1000 - 15 = 985$ stars.

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So $1 + 2 + 3 + 4 + 5 = 15$ stars already placed.

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$\frac{989!}{4!985!}$. This is also $\binom{989}{4}$. Discuss.

Strange Poker-two 3-of-a-kind

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$$\frac{\binom{r}{2}\binom{s}{3}^2}{\binom{rs}{6}}.$$

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Four of a kind: Pick a rank r.

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2) What is the probability of getting four-of-a-kinds?
Four of a kind: Pick a rank r. Pick 4 suits $\binom{s}{4}$.

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2) What is the probability of getting four-of-a-kinds?
Four of a kind: Pick a rank r. Pick 4 suits $\binom{s}{4}$.
Pick 2 more cards but they can't be of rank r.

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2) What is the probability of getting four-of-a-kinds?
Four of a kind: Pick a rank r. Pick 4 suits $\binom{s}{4}$.
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If you didn't know to not use the same rank, I will still mark it right.