

**HW 10 CMSC 452. Morally DUE Nov 18**

NOTE- THIS HW IS TWO PAGES LONG.

1. (40 points) (Recall that  $\#_\sigma(w)$  means the number of  $\sigma$  in  $w$ .)
- (a) Assume the alphabet is  $\Sigma = \{a, b, c, d\}$ . Write a Context Sensitive Grammar for the language

$$\{w \mid \#_a(w) = \#_b(w) = \#_c(w) = \#_d(w)\}.$$

- (b) Let  $n \in \mathbf{N}$ . Assume the alphabet is  $\Sigma = \{1, 2, 3, 4, \dots, n\}$ . Write a Context Sensitive Grammar for the language

$$\{w \mid \#_1(w) = \#_2(w) = \dots = \#_n(w)\}.$$

(You may use DOT DOT DOT.)

2. (30 points) RECALL: as part of the proof that  $L_n$  has a large CFL we proved the following (I am stating it differently.)

*If  $T$  is a binary tree (every node has  $\leq 2$  children) with  $n$  leaves then there is a node with  $L$  leaves below it where  $n/3 \leq L \leq 2n/3$ .*

In THIS problem we generalize this.

- (a) Find constants  $\alpha$  and  $\beta$  with  $0 < \alpha < \beta < 1$  such that the following holds. (And PROVE your result.)

*If  $T$  is a trinary tree (every node has  $\leq 3$  children) with  $n$  leaves then there is a node with  $L$  leaves below it where  $\alpha n \leq L \leq \beta n$ .*

- (b) Let  $k$  be a constant. Find constants  $\alpha$  and  $\beta$  with  $0 < \alpha < \beta < 1$  ( $\alpha$  and  $\beta$  will depend on  $k$ ) such that the following holds. (And PROVE your result.)

*If  $T$  is a  $k$ -ary tree (every node has  $\leq k$  children) with  $n$  leaves then there is a node with  $L$  leaves below it where  $\alpha n \leq L \leq \beta n$ .*

3. RECALL from the Thursday Nov 6 lecture (or the notes on Context Sensitive Langs on the class webpage)

- $L_n$  is the set of all permutations of  $\{1, 2, 3, \dots, n\}$ . (That was in Theorem 2.4 in the notes.) (For example  $L_3 = \{123, 132, 213, 231, 312, 321\}$ .)

- A CSG  $G = (N, \Sigma, S, R)$  is in *Chomsky Normal Form* if every rule in  $R$  is either of the form  $A \rightarrow CD$  OR  $AB \rightarrow CD$  OR  $A \rightarrow \sigma$  where  $A, B, C, D \in N$  and  $\sigma \in \Sigma$ .

NOW here is the problem I want you to solve: Give a CHOMSKY NORMAL FORM CSG for  $L_n$ .

**IF YOU GOT  $\leq 60$  ON THE MIDTERM!!**

You can submit the midterm itself together with the problems you got wrong done RIGHT but with very clear explanations, and neat. This will NOT add to your score but it WILL count for *consideration* if after the final you are on the C-D border. (It will not help at all on the B-C or A-B border.) You need to do this by NOVEMBER 18. (NO dead-cat extension allowed.) This document should be SEP from the HW.

ALSO, if you weren't going to class, goto class. If you weren't doing the hw on the moral deadline DO the HW on the moral deadline.