

HW 12 CMSC 452. Morally DUE Dec 2

NOTE- THIS HW IS TWO PAGES LONG

1. (0 points) When is the final? (Hint- Look at the Course Webpage under Policy.)
2. (25 points) Let 3-SAT be the set of all formulas in CNF-SAT that have EXACTLY 3 literals per clause. Show that $\text{CNF-SAT} \leq_m^p \text{3-SAT}$. (This is a well known result and you CAN look it up in your text or on the web. However, hand it in in your own words and name your source.)
3. (25 points) Let DNF-SAT be the set of all satisfiable formulas of the form $D_1 \vee D_2 \vee \dots \vee D_m$ where each D_i is an AND of literals. Prove EITHER (a) DNF-SAT is in P, or (b) DNF is NP-complete. (IF you look it up (not sure that will work) then name your source.)
4. (25 points) In this problem all graphs are undirected. Let

$$\text{HAMCY} = \{G \mid G \text{ Has a Hamiltonian Cycle} \}.$$

$$\text{HAMPA} = \{G \mid G \text{ Has a Hamiltonian Path} \}.$$

For each of the following say if its TRUE, FALSE, or UNKNOWN TO SCIENCE. In all cases explain.

(a) $\text{HAMPA} \leq_m^p \text{HAMCY}$.

(b) $\text{HAMCY} \leq_m^p \text{HAMPA}$.

5. (25 points) Let $M_1, M_2, \dots$ be a standard list of Turing Machines. (You must do this problem WITHOUT Rice's theorem. If you don't know what Rice's theorem is don't worry about it.)

(a) Let

$$A = \{x \mid \text{for every prime } y, M_x(y) \downarrow\}$$

(NOTE- if $x \in A$ then we know that M_x halts on every prime; however, we do NOT know what it does on non-primes.)

Show that A is UNDECIDABLE.

- (b) Let Y be any non-empty subset of the naturals.
Let

$$A = \{x \mid \text{for every } y \in Y, M_x(y) \downarrow\}$$

(NOTE- if $x \in A$ then we know that M_x halts on every element of Y ; however, we do NOT know what it does on non-elements-of- Y .)

Show that A is undecidable.

- (c) Let

$$A = \{x \mid \text{for every } y \in Y, M_x(y) \uparrow\}$$

Show that A is undecidable.