452, FINAL

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **No calculators or other aids are allowed**. If you have a question raise your hand.
2. There are 5 problems which add up to 100 points. The exam is 2 hours.
3. There are 7 pages. Make sure you have 7 pages!
4. To get some partial credit show all of your work for each problem, **write legibly**, and **clearly indicate** your answers. **NO** Credit for illegible answers.
5. After the last page there is paper for scratch work. If you need extra scratch paper, raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
6. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

7. Fill in the following:

NAME :
SIGNATURE :
SID :
SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
Prob 5:	
TOTAL	

1. (20 points) For this problem you may assume $P \neq NP$. Give an example of each of the following. NO PROOF REQUIRED

   (a) A language that is REGULAR but NOT FINITE.

   (b) A language that is CONTEXT FREE but NOT REGULAR.

   (c) A language that is P but NOT CONTEXT FREE.

   (d) A language that is NP but NOT P.

   (e) A language that is C.E. but NOT DECIDABLE.

   (f) A language that is NOT C.E.

2. (20 points) Fill in the blanks. NO justification needed. The $XXX(n)$, $YYY(n_1, n_2)$, $ZZZ(n_1, n_2)$ are functions of the indicated variables.

   (a) For all L, for all n, if L is accepted by an NDFA with n states then L is also accepted by a DFA with $XXX(n)$ states. (Make $XXX(n)$ as small as possible.)

   (b) For all L_1, L_2, n_1, n_2 if L_1 is accepted by a DFA with n_1 states and L_2 is accepted by a DFA with n_2 states then $L_1 \cap L_2$ is accepted by a DFA with $YYY(n_1, n_2)$ states. (Make $YYY(n)$ as small as possible.)

   (c) For all L_1, L_2, n_1, n_2 if L_1 is generated by a CFG with n_1 nonterminals and L_2 is generated by a CFG with n_2 nonterminals then $L_1 \cup L_2$ is generated by a CFG with $ZZZ(n_1, n_2)$ nonterminals. (Make $ZZZ(n)$ as small as possible.)

3. (20 points) Give a description of the algorithm that takes a CFG G in Chomsky Normal form, and a string w, and determines if $w \in L(G)$. (The algorithm using Dynamic programming, not the brute force algorithm.) It must be so clear that someone who does not know it will understand WHY it works and be able to code it up (but won't want to).

4. (20 points) Let $A_1, A_2 \in NP$. Let p_1, p_2, be polynomials and $B_1, B_2 \in P$ such that

$$A_1 = \{x \mid (\exists y)[|y| = p_1(|x|) \text{ AND } (x, y) \in B_1\}.$$

$$A_2 = \{x \mid (\exists y)[|y| = p_2(|x|) \text{ AND } (x, y) \in B_2\}.$$

Prove that $A_1 \cap A_2 \in NP$.

5. Assume that $P = NP$. Let $A \in NP$. Let p be a polynomial and $B \in P$ be such that

$$A = \{x \mid (\exists y)[|y| = p(|x|) \text{ AND } (x, y) \in B\}.$$

Show that the following FUNCTION can be computed in polynomial time.

$f(x)$ is

(a) NO if $x \notin A$.

(b) (YES, y) if $x \in A$ and $(x, y) \in B$. (That is, f reports YES and also produces the witness y.)

Scratch Work