

Project 1: WS1S Project Morally DUE Oct 7

(Usual rules- can hand it in on Oct 9 with no penalty, say if your cat died.)

1. (0 points) What is your name? Write it clearly. Staple your HW.
2. (This is not a problem, this is a summary of what you are doing and why.) In this project we will go through the WS1S Decidability algorithm to see if the following sentence is decidable:

$$(\exists A)(\forall x)[x \in A \rightarrow x \equiv 1 \pmod{2}]$$

The point is to go through the decision procedure, so even if you think you know if its true or false (and you are probably right), I want you to FOLLOW THE PROCEDURE.

3. (10 points) Rewrite it the formula in the form:

$$(\exists A)\neg(\exists x)\neg[\phi_1(x, A) \vee \phi_2(x, A)]$$

and then in the form:

$$(\exists A)\neg(\exists x)[\psi_1(x, A) \wedge \psi_2(x, A)]$$

(NOTE- ψ_1 or ψ_2 might have negations in them, so could be of the form $\neg\alpha$. In the later parts where I ask you to build a DFA for the truth set of ψ_1 or ψ_2 you might want to build one for the truth set of α and then swap accept and dignified-reject states. This is a matter of taste.)

NOTE- it may well be that $\psi_1(x, A)$ or $\psi_2(x, A)$ does not depend on one of the free variables shown. Even so, we must treat them as being on those two vars for the construction.)

4. (10 points) Write a DFA that accepts $\{(x, A) \mid \psi_1(x, A)\}$
5. (10 points) Write a DFA that accepts $\{(x, A) \mid \psi_2(x, A)\}$
6. (10 points) Write a DFA that accepts $\{(x, A) \mid \psi_1(x, A) \wedge \psi_2(x, A)\}$.
(USE the cross product construction on the last two problems.)

7. (10 points) Write an NFA that accepts $\{A \mid (\exists x)[\psi_1(x, A) \wedge \psi_2(x)]\}$.
(This is closure under projection. Just erase stuff from the last problem.)
8. (15 points) Convert the NFA in the last problem to a DFA.
(HINT: Use the usual construction of NFA to DFA, but you do not need all of the states in the powerset. Just use sets of states as you need them.)
9. (15 points) Write a DFA for $\{A \mid \neg(\exists x)[\psi_1(x, A) \wedge \psi_2(x)]\}$.
10. (15 points) Let M be the DFA in the last problem. Give five strings that it accepts.
11. (15 points) Since there were strings that M accepts, what does that say about our original formula? What does that say about the five strings from the last problem?